理想斜压波系统中垂直涡度和水平散度的中尺度谱

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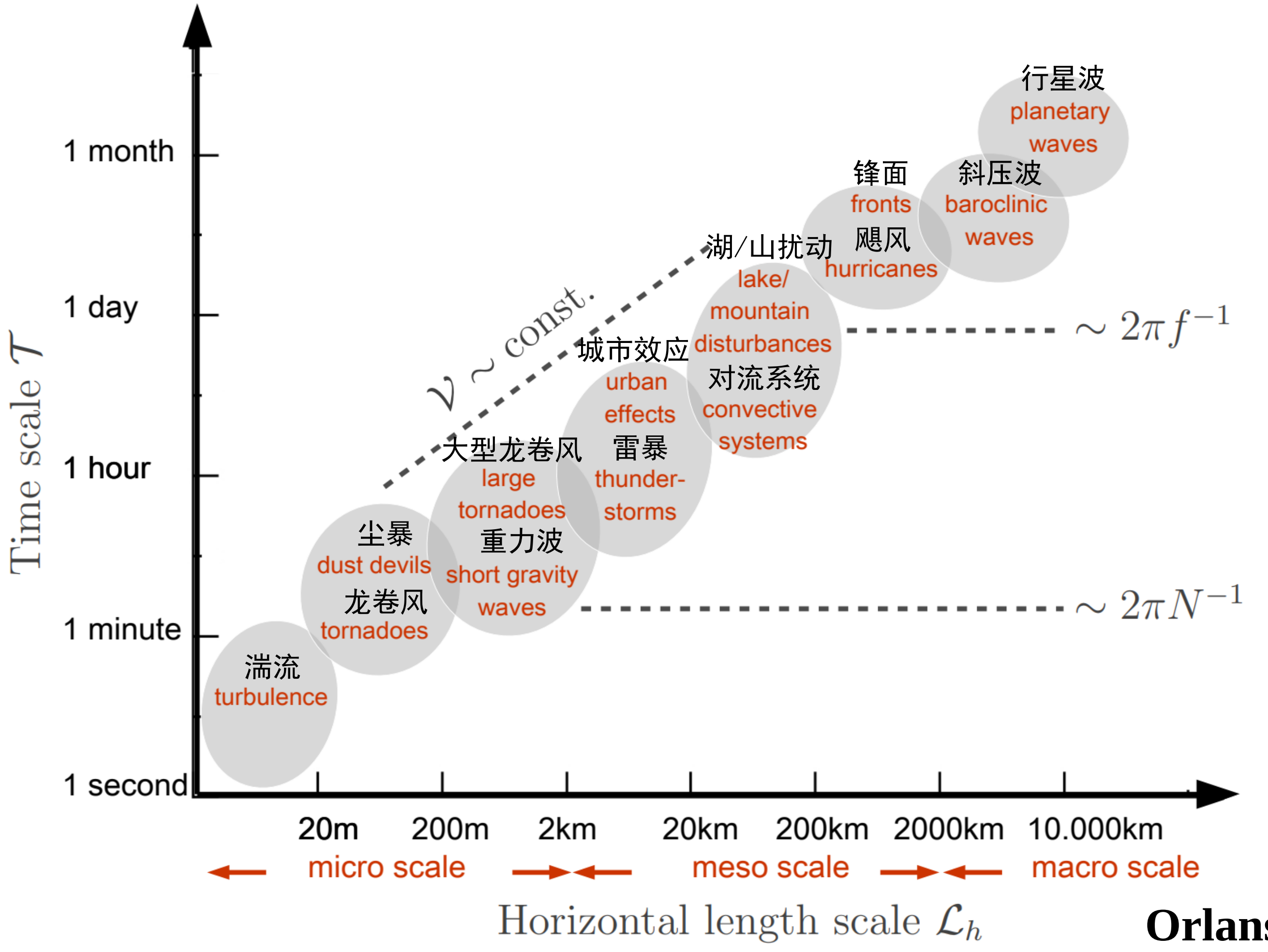
Outlines

1. Introduction
2. Methodology and Idealized Simulation
3. Vorticity/divergence Spectra and Spectral budgets
4. Associating spectral energy and SV/SD transfers
5. Summary

1. Introduction

大气运动的多时空尺度特征

地球大气是一个非线性的复杂动力系统，其中各种各样具有不同空间和时间尺度的动力和热力过程互相影响。



Orlanski, 1975

在**天气尺度**上，**对应数百到数千公里的尺度范围**，大气运动以准地转涡旋为主，即气压梯度力和科氏力近似平衡，且主要受斜压不稳定驱动；

在**小尺度**上，**对应数公里以下的尺度范围**，大气运动具有显著的辐散非平衡特征，且主要受对流不稳定驱动；

而在**二者之间的中间尺度（即中尺度）**上，大气运动存在多种不稳定机制（例如惯性不稳定、对称不稳定、切变不稳定），且准地转涡旋平衡运动和辐散非平衡运动通常都不占主导，具有明显的非地转特征

1. Introduction

能量谱

- 大气运动总是伴随着大气能量的产生、传播或者耗散过程，从能量学角度研究大气运动规律是大气科学研究的一种有效方法
- 中尺度联系着包含主要能量的天气尺度与湍流耗散发生的小尺度，因此中尺度范围上能量串级过程显著，体现了大气运动的多尺度非线性本质
- 大气能量作为描述大气状态的物理量，对其进行谱分析可以为研究大气多尺度非线性问题提供新的物理视角

能量谱（**Energy spectrum**），即大气能量随水平波数（或尺度）的函数分布

方差谱（**Variance Spectrum**）或者功率谱（**Power spectrum**）

1. Introduction

能量谱的计算

- 谱计算采用FFT 展开，即二维离散傅立叶变换，记

$$\hat{\varphi}(\mathbf{k}) = FFT(\varphi) \qquad \mathbf{k} = (k_x, k_y)$$

$$(a,\ b)_{\mathbf{k}} = \hat{a}(\mathbf{k})\hat{b}(\mathbf{k}) \quad (\mathbf{a},\ \mathbf{b})_{\mathbf{k}} = \hat{\mathbf{a}}(\mathbf{k})\cdot\hat{\mathbf{b}}(\mathbf{k})$$

- 能量谱的定义

$$E_h(\mathbf{k}) = (\mathbf{u},\ \mathbf{u})_{\mathbf{k}}/2 \quad E_z(\mathbf{k}) = (w,\ w)_{\mathbf{k}}/2$$

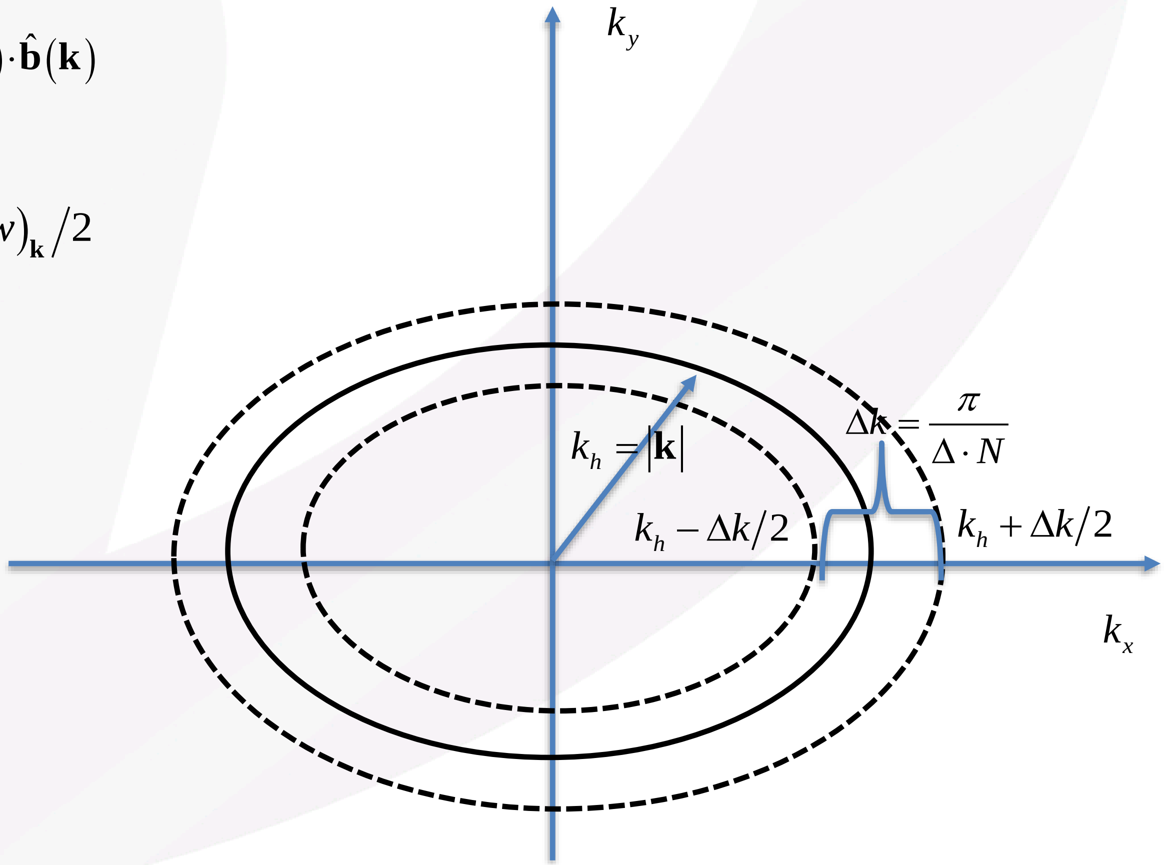
$$E_A(\mathbf{k}) = \gamma(z)(\theta'_m,\ \theta'_m)_{\mathbf{k}}/2$$

- 一维水平波数谱的构造

$$k_h = |\mathbf{k}| = \sqrt{k_x^2 + k_y^2}$$

$$\Delta k = \frac{\pi}{\Delta \cdot N} \qquad N = \min(N_i, N_j)$$

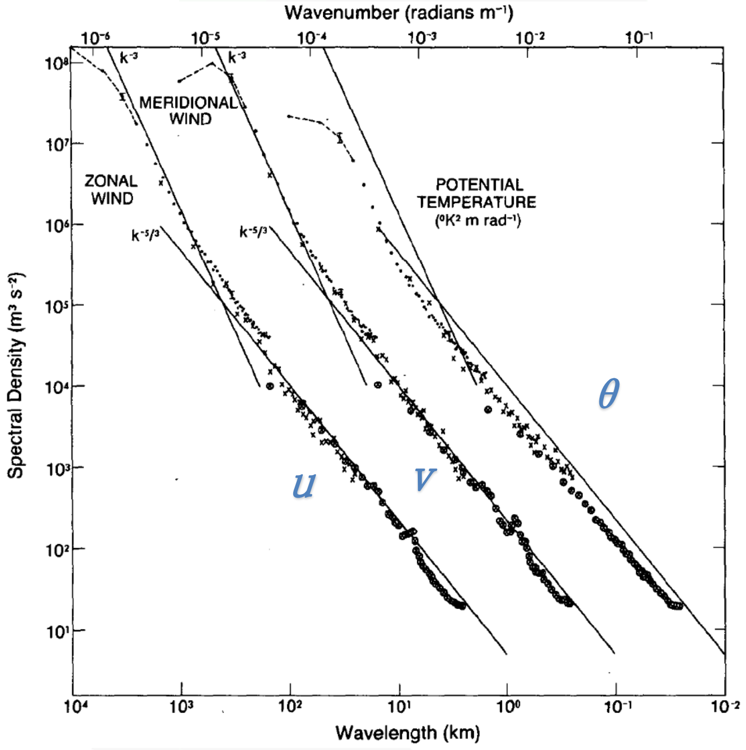
$$E_*(k_h) = \sum_{k_h - \Delta k/2 \le |\mathbf{k}| < k_h + \Delta k/2} E_*(\mathbf{k})/\Delta k$$



能量谱的基本特征

观测：Nastrom and Gage-Spectrum，简称N-S谱

纬向风、经向风速和温度，每个量偏移1个量级



Nastrom and Gage (1985 JAS)

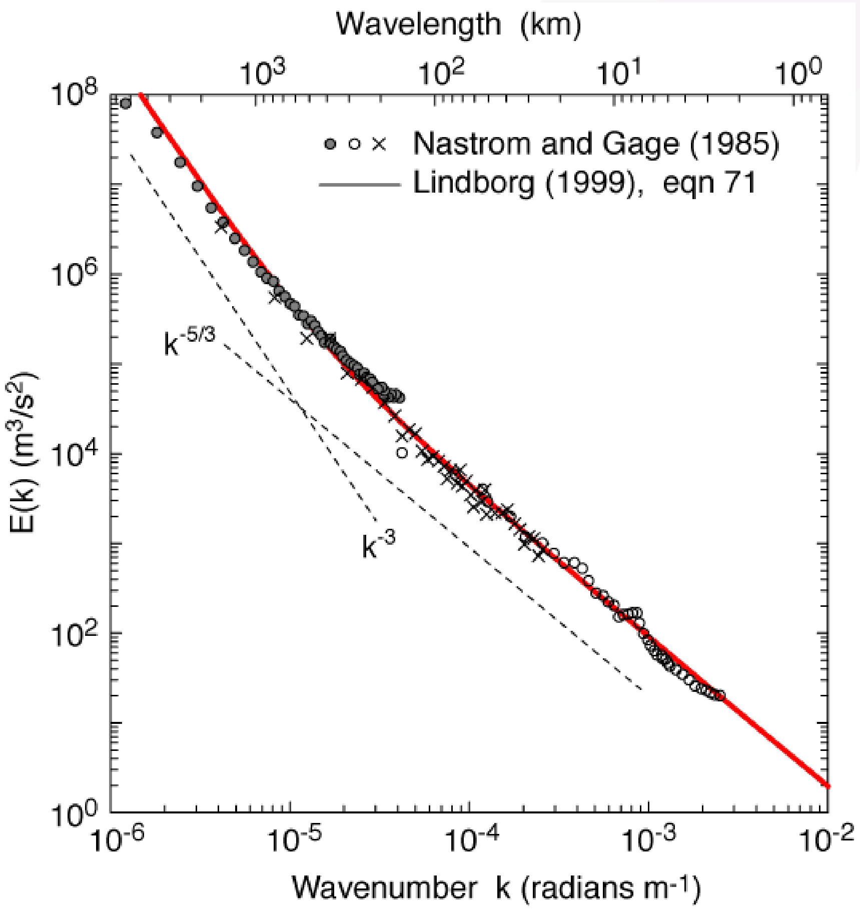
商用飞机风速和温度测量

飞机高度在对流层顶附近

水平分辨率2.5 km：

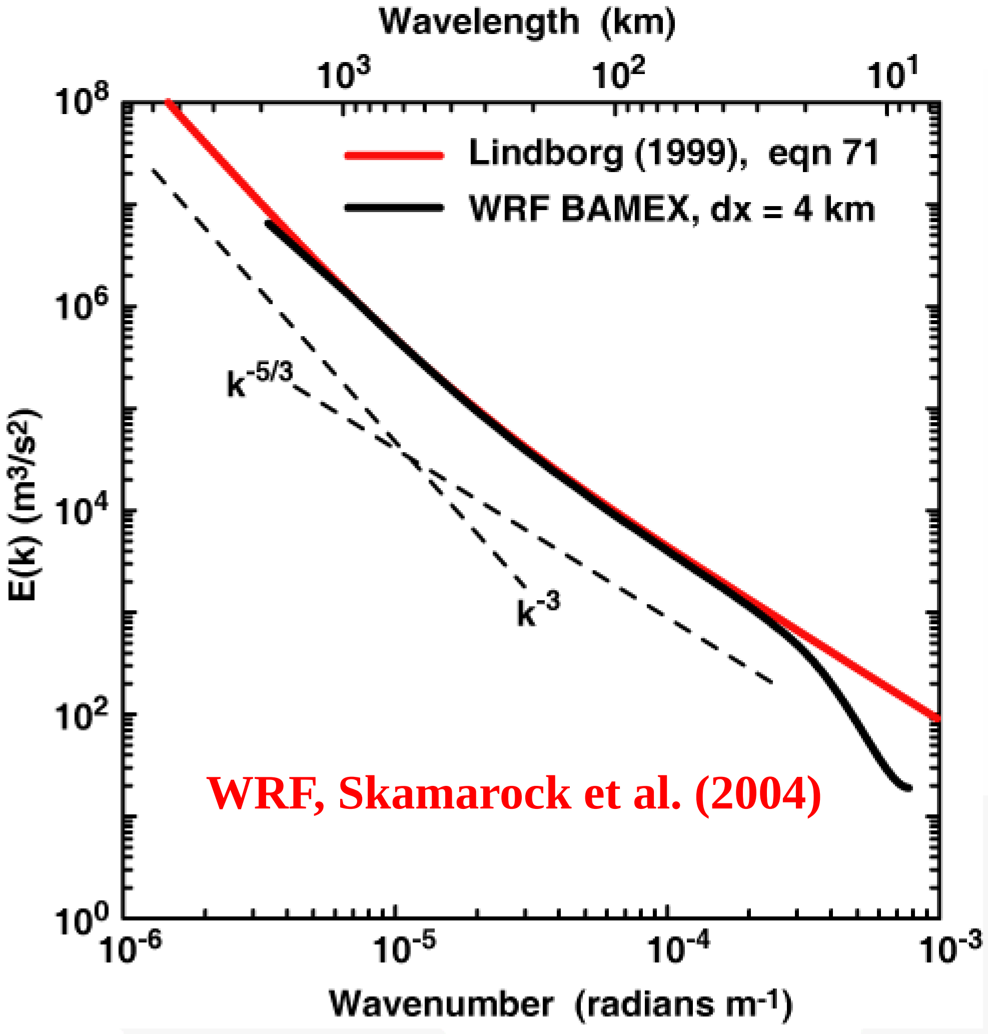
- ✓ 中尺度-5/3谱达到400 km
- ✓ 天气尺度-3谱

-3斜率形成的机制
二维湍流理论（Kraichnan 1967)
准地转理论 (Charney 1971)

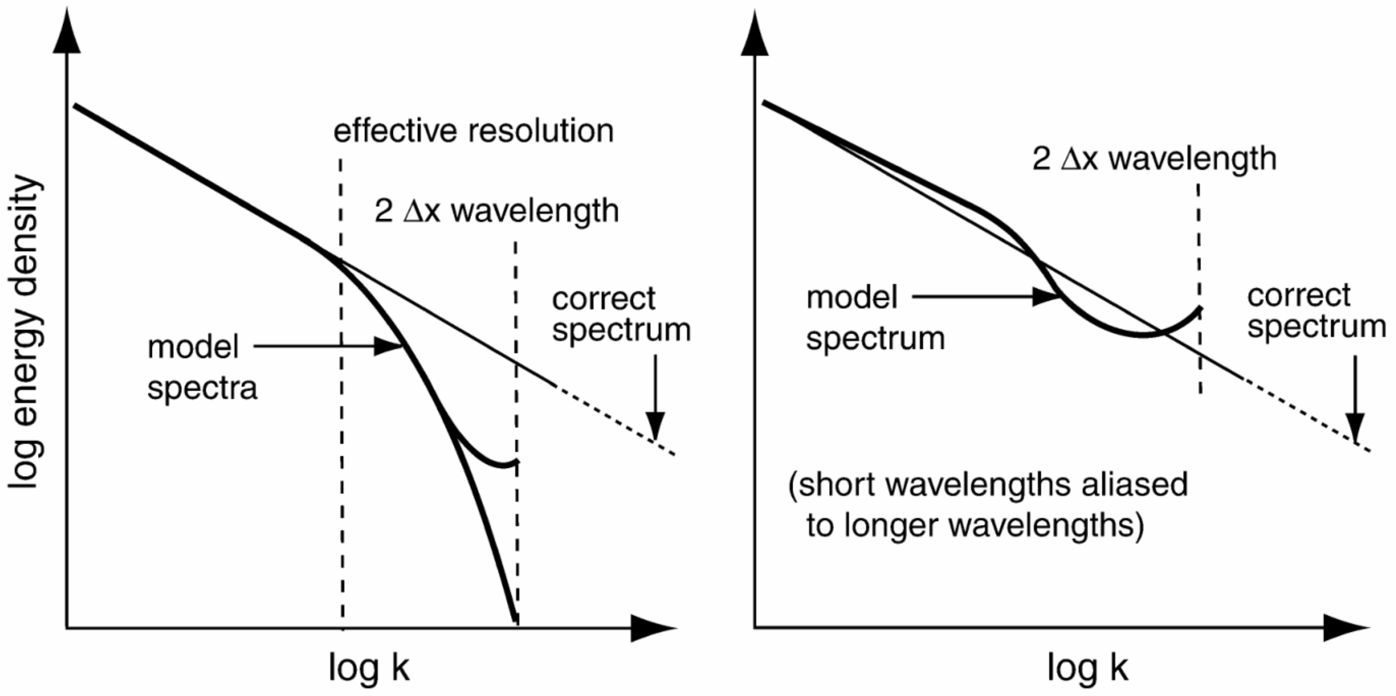


1. Introduction

数值模式的有效分辨率



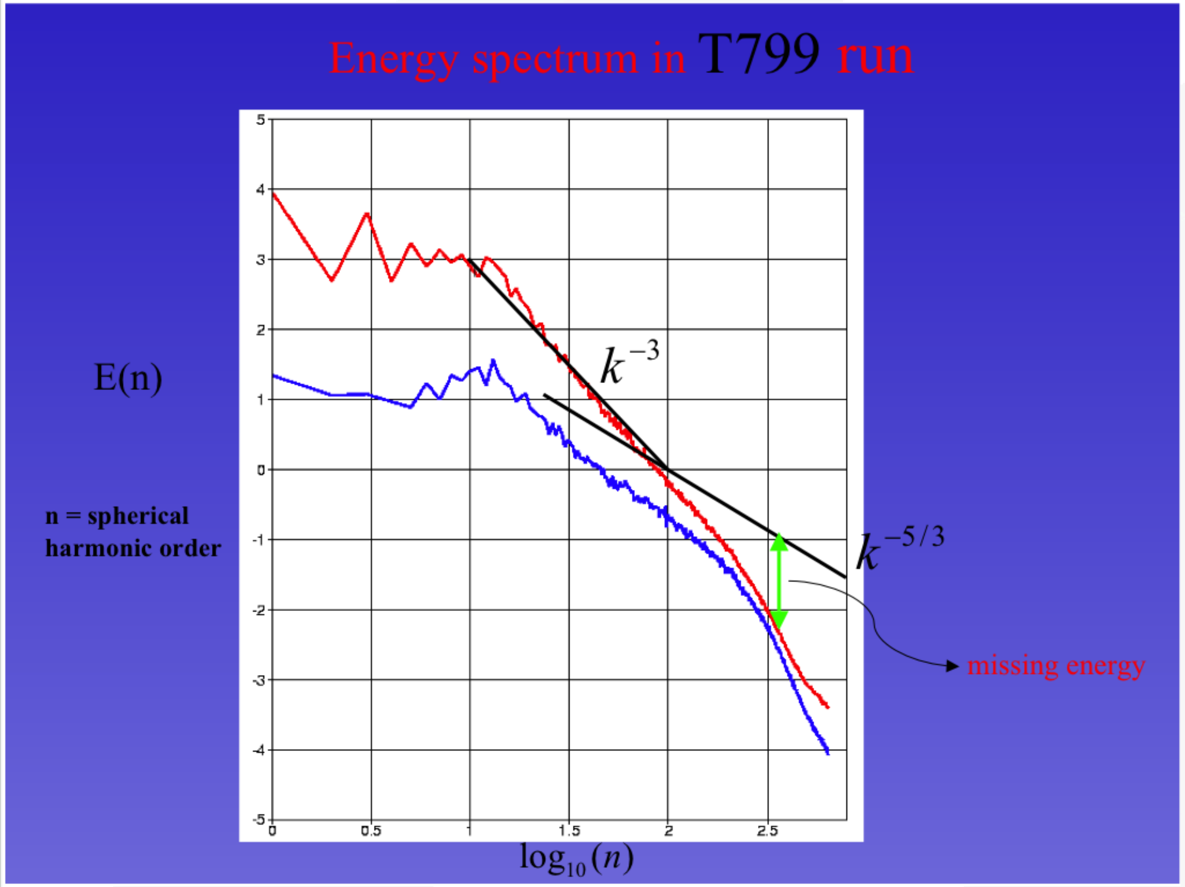
两类模式大气能量谱示意图



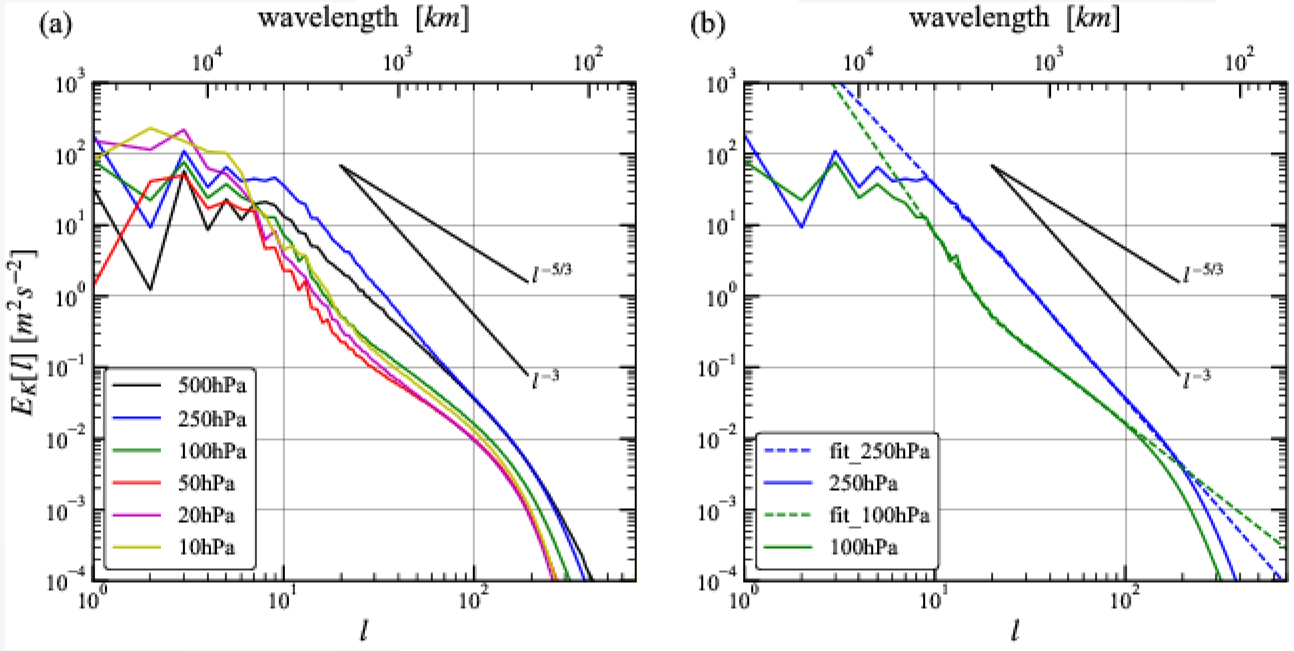
- 定量揭示不同耗散方案的滤波效应；
- 合理设置耗散方案中的滤波系数；

1. Introduction

数值模式的有效分辨率



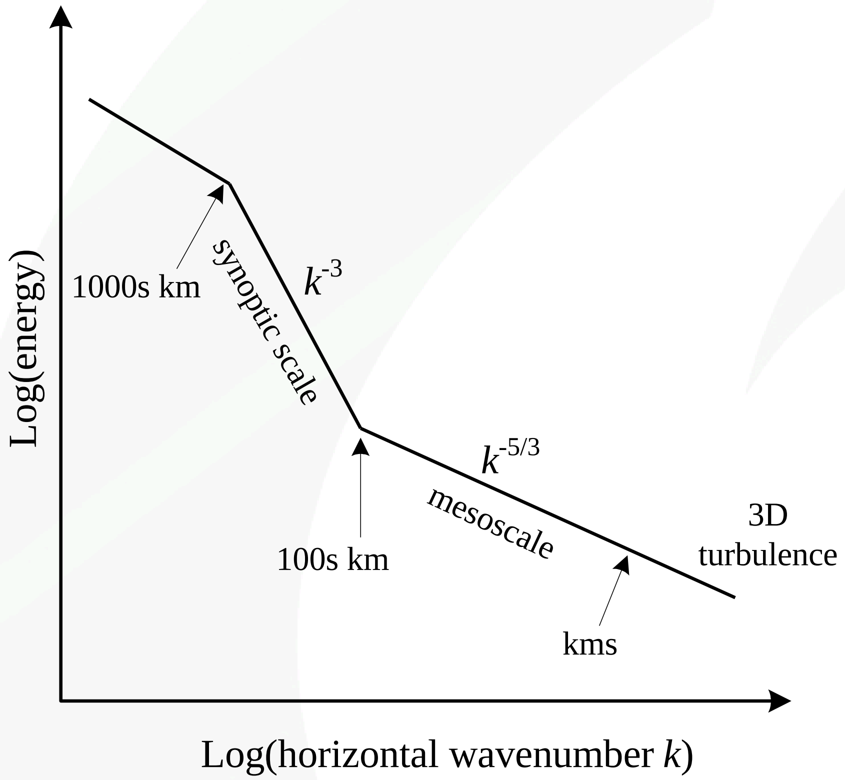
- ECWMF的IFS全球中期数值天气预报最高水平的代表，却无法合理模拟出中尺度-5/3特征。
  - ✓ 暗示其中尺度能量偏弱？
  - ✓ 原因是什么？天气尺度耗散过强？
  - ✓ 多大程度上影响了其中尺度模拟预报水平？



Li and Peng (JAS, 2023)

1. Introduction

中尺度可预报性



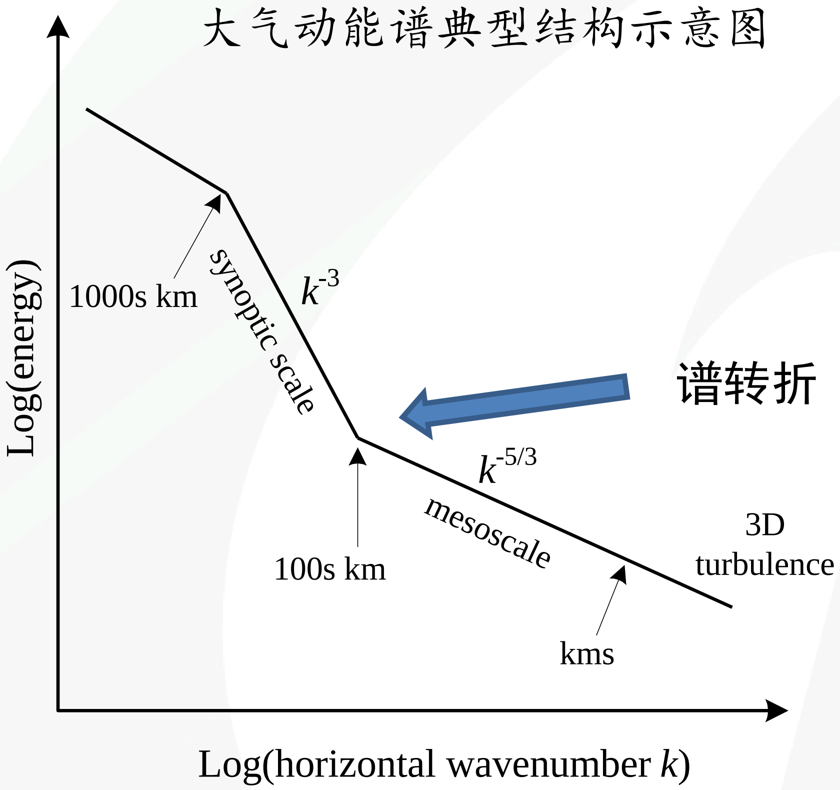
经典湍流理论:

k^{-3}**流型具有无限可预报性，** $k^{-5/3}$**流型具有有限的可预报性**

- **决定中尺度$k^{-5/3}$的关键过程是什么？对中尺度可预报的暗示是什么？例如湿物理过程？**
- **模式大气复制中尺度-5/3斜率特征的能力与其中尺度可预报性的关系？** (Durran et al., 2014; Durran et al. 2016)
- **大气中尺度可预报性与背景大气能量谱特征是否有关系？** (Lloveras et al. 2021)

1. Introduction

能量谱的典型结构



1000s km泛指数千公里，100s km泛指数百公里，kms泛指数公里

Skamarock et al. (2014, JAS)

天气尺度-3斜率形成的机制：二维湍流理论（Kraichnan 1967)或准地转理论 (Charney 1971)

中尺度-5/3斜率特征的动力学解释还没有形成统一的认知，是大气中尺度动力学研究领域的前沿科学问题（Lindborg 2005, 2007; Tulloch and Smith 2009; Waite and Snyder 2013; Peng et al. 2014a, 2015a; Sun and Zhang 2017; Wang et al. 2018； Menchaca and Durran 2019; Ambacher and Waite 2020; Li et al. 2023;）。

1. Introduction

中尺度能量谱理论研究进展

中尺度能量串级的性质和强迫仍然存在争议，但最近的证据倾向于降尺度串级

升尺度串级

（能量从小尺度串级到大尺度）

Upscale cascade, inverse cascade

从小尺度（例如对流）强迫的平衡运动

2D湍流理论（Gage 1979;Lilly 1983;Vallis et al. 1997 ）

准地转理论 能量降尺度串级

准地转的能量和涡度拟能的双串级理论（Tung and Orlando 2003, Pouquet et al. 2017)

对流层顶附近的准地转动力学（Tulloch and Smith 2006, 2009; Asselin et al. 2018）

降尺度串级

（能量从大尺度串级到小尺度）

Downscale cascade, direct cascade

由斜压波激发的非平衡运动（例如重力波）

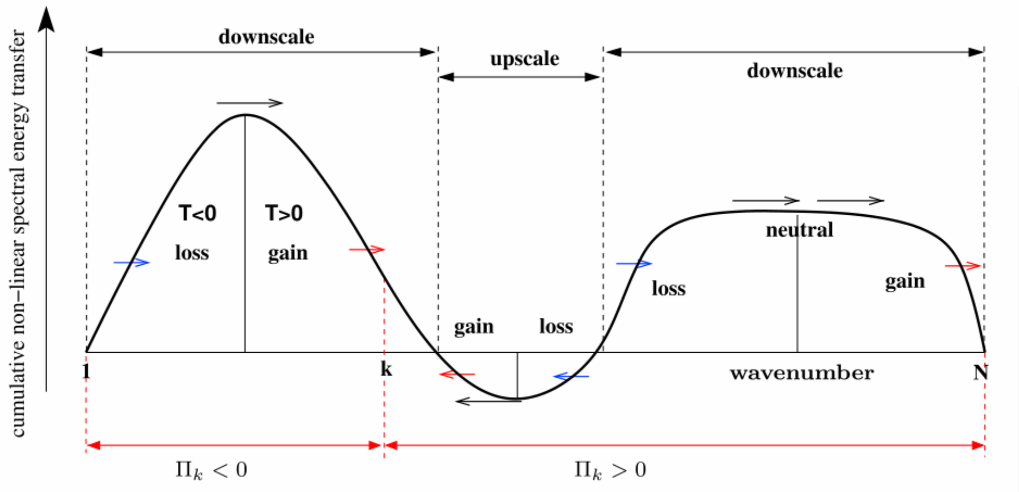
惯性重力波（IGWs; Dewan 1979; Bühler et al. 2014; Callies et al. 2014, 2016; Kafiabad et al. 2019）

层结湍流/旋转层结湍流（Lindborg 2006; Riley and Lindborg 2008; Waite and Bartello 2006; Bartello 2010）

自激发的非平衡运动（Kafiabad and Bartello 2018）

1. Introduction

中尺度能量谱理论研究的局限



- 以往的串级理论通常假定**大气中尺度为惯性子域**，即不存在显著的能量源/汇
- 实际大气中尺度上存在丰富的**间歇性直接物理强迫**，例如湿物理过程、地形效应等

- **湿物理及其相关过程对大气中尺度能量谱形成的作用?**
- **这些直接强迫作用对大气中尺度能量串级、转化、垂直输送的影响？等等**

1. Introduction

基于数值模式的模拟研究

- 可以弥补观测资料时空分辨率的不足
- 定量评估一些不能直接观测的物理量，例如潜热加热、耗散等
- 可以针对研究目的灵活设计敏感性试验

基于复杂实际大气数值模式的模拟诊断成为了研究大气能量谱特征及其形成机理的有效手段；能量谱及其理论研究也为数值模式提供了定量化动力学评估检验。

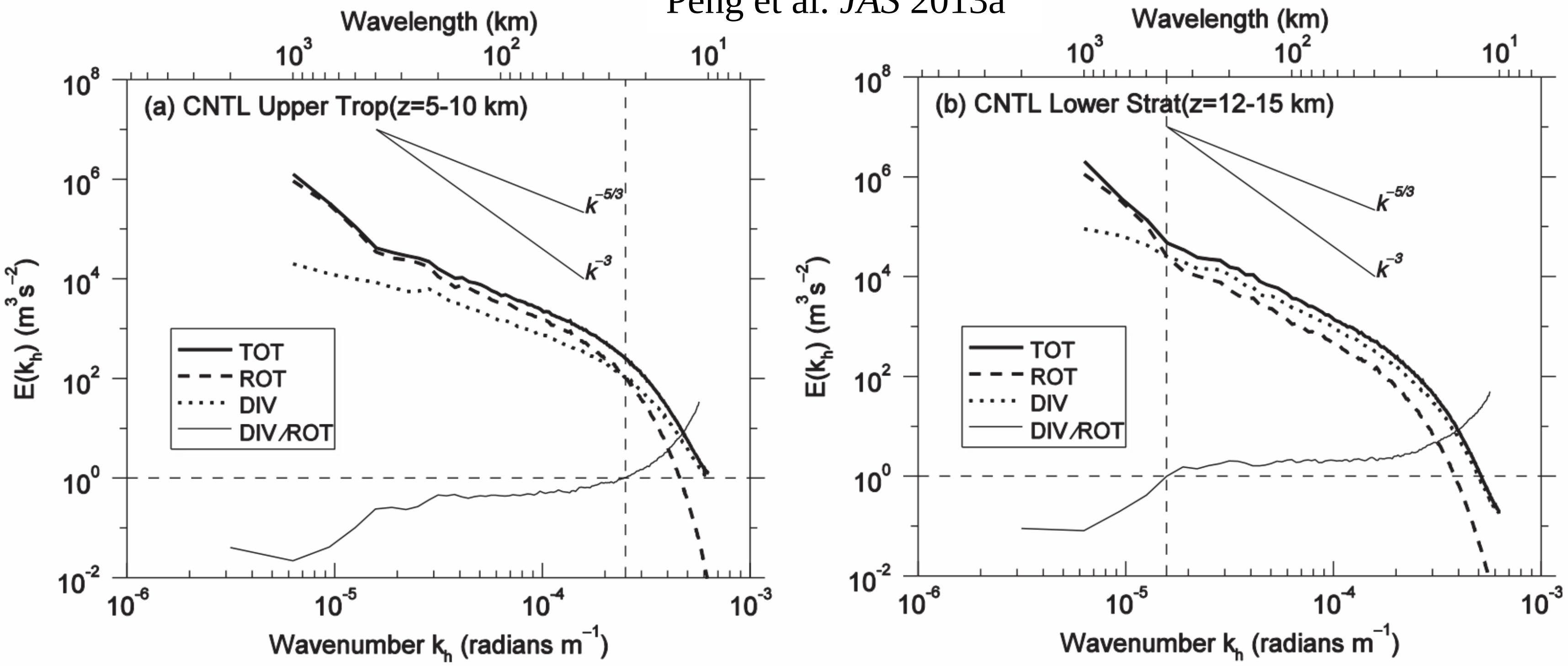
- ✓ 这些分析研究表明：**中尺度动能谱不太可能单纯是由串级过程产生的，而是几种不同机制在不同高度或尺度上共同作用的结果**（Waite and Snyder 2009, 2013；Peng et al. 2014a, b, 2015a, b；Sun et al. 2017；Wang et al. 2018；Li and Peng 2023; Peng et al. 2023）。
- ✓ 不同学者针对几种典型中尺度系统（例如**梅雨锋系统**、**斜压波系统**、**飑线**等）开展了理想数值模拟，发现垂直传播重力波和湿对流本身的重要性，指出升/降尺度串级同时存在的可能性。
- ✓ 不同数值模式在表征中尺度能量谱上还存在一定的分歧。

1. Introduction

中尺度能量谱中的波涡贡献

- Decomposing mesoscale spectra into wave and vortex contributions is the key to evaluate the various proposed theories

Peng et al. *JAS* 2013a



- Helmholtz decomposition

$$E_K(\mathbf{k}) = \frac{1}{2}(\mathbf{u},\mathbf{u})_{\mathbf{k}} = \frac{1}{2}\left[\hat{u}^*(\mathbf{k})\hat{u}(\mathbf{k}) + \hat{v}^*(\mathbf{k})\hat{v}(\mathbf{k})\right]$$

$$E_{RK}(\mathbf{k}) = \frac{1}{2}(\mathbf{u}_R,\mathbf{u}_R)_{\mathbf{k}} = \frac{1}{2}\frac{(\zeta,\zeta)_{\mathbf{k}}}{|\mathbf{k}|^2}$$

$$E_{DK}(\mathbf{k}) = \frac{1}{2}(\mathbf{u}_D,\mathbf{u}_D)_{\mathbf{k}} = \frac{1}{2}\frac{(\delta,\delta)_{\mathbf{k}}}{|\mathbf{k}|^2}$$

- Geostrophic/ageostrophic decomposition

- The RKE spectrum slightly dominates over DKE spectrum in the tropospheric mesoscale (Cho et al. 1999; Lindborg 2007)
- The DKE spectrum actually dominates over the RKE spectrum in the lower-stratospheric mesoscale (Callies et al. 2016; Li and Lindborg 2018).

1. Introduction

中尺度能量谱中的波涡贡献

➢ 能量（Energy）谱 VS 拟能（Enstrophy）谱

$$E_K(\mathbf{k}) = \frac{1}{2}(\mathbf{u},\mathbf{u})_{\mathbf{k}} = \frac{1}{2}\left[\hat{u}^*(\mathbf{k})\hat{u}(\mathbf{k}) + \hat{v}^*(\mathbf{k})\hat{v}(\mathbf{k})\right]$$

$$E_{RK}(\mathbf{k}) = \frac{1}{2}(\mathbf{u}_R,\mathbf{u}_R)_{\mathbf{k}} = \frac{1}{2}\frac{(\zeta,\zeta)_{\mathbf{k}}}{|\mathbf{k}|^2}$$

$$E_{DK}(\mathbf{k}) = \frac{1}{2}(\mathbf{u}_D,\mathbf{u}_D)_{\mathbf{k}} = \frac{1}{2}\frac{(\delta,\delta)_{\mathbf{k}}}{|\mathbf{k}|^2}$$

$$E_K(\mathbf{k}) = E_{RK}(\mathbf{k}) + E_{DK}(\mathbf{k})$$

$$\hat{\zeta}(\mathbf{k}) = ik_x\hat{v}(\mathbf{k}) - ik_y\hat{u}(\mathbf{k})$$

$$\hat{\delta}(\mathbf{k}) = ik_x\hat{u}(\mathbf{k}) + ik_y\hat{v}(\mathbf{k})$$

$$\Phi(\mathbf{k}) = \frac{1}{2}(\zeta,\zeta)_{\mathbf{k}}$$

$$\Psi(\mathbf{k}) = \frac{1}{2}(\delta,\delta)_{\mathbf{k}}$$

$$\Phi(\mathbf{k}) + \Psi(\mathbf{k}) = |\mathbf{k}|^2 E(\mathbf{k})$$

- At planetary and synoptic scales, the atmospheric motion is dominated by the QG dynamics. Thus the enstrophy (i.e., half of the squared vertical vorcitity) is conserved and its cascade has direct implications for energy cascade. Downscale enstrophy cascade (Kraichnan 1967)
- At mesoscale, what the spectral enstrophy flux would be like? To what extent the spectral energy flux is linked to the spectral enstrophy flux? Or whether there are other non-conservated processes that dominate the spectral energy flux?

Important: The equal importance of divergent and rotational components of the mesoscale atmosphere flows !!!

1. Introduction

研究目的

SV= half of the squared vertical vorticity
SD= half of the squared horizontal divergence

- **Explore the dynamics underlying the mesoscale spectra of divergent and rotational motion components**
- Focusing on SV and SD or on RKE and DKE is both relevant to the subject.

Why we switch to the spectral budget of SV and SD?

- **Initial Motivation:** mathematically cumbersome in constructing the conserved spectral flux terms of RKE and DKE; however, it is very convenient to construct the conserved spectral fluxes of SV and SD;

$$\Phi(\mathbf{k}) + \Psi(\mathbf{k}) = |\mathbf{k}|^2 E(\mathbf{k})$$

- **Motivation 2:** the spectral budget of SV and SD naturally highlights the contribution of smaller scales
- **Motivation 3:** explore possible mesoscale enstrophy cascade under the nonlinear advection of the full horizontal velocity
- **Motivation 4:** to explicitly associate spectral energy and SV/SD transfers, which provides additional physical views on the mesoscale energy cascade.

2. Methodology and Idealized Simulation

涡度/散度谱收支方程推导

f 平面一般湿大气水平动量方程

$$\partial_t \mathbf{u} = -(\mathbf{u}\cdot\nabla\mathbf{u} + w\partial_z\mathbf{u}) - c_p\bar{\theta}\nabla\pi' - f\mathbf{e}_z\times\mathbf{u} + \mathcal{D}_\mathbf{u}$$

$$\partial_t\zeta = -(\mathbf{u}\cdot\nabla)\zeta - w\partial_z\zeta - f\delta - \zeta\delta - \mathbf{e}_z\cdot\nabla w\times\partial_z\mathbf{u} + \mathbf{e}_z\cdot\nabla\times\mathcal{D}_\mathbf{u}$$

$$\partial_t\delta = -(\mathbf{u}\cdot\nabla)\delta - w\partial_z\delta + f\zeta - \left(\delta^2 - 2J(u,v)\right) - c_p\bar{\theta}\nabla^2\pi' - \nabla w\cdot\partial_z\mathbf{u} + \nabla\cdot\mathcal{D}_\mathbf{u}$$

$$\partial_t\zeta = -(\mathbf{u}\cdot\nabla)\zeta - \zeta(\nabla\cdot\mathbf{u})/2 + \zeta(\nabla\cdot\mathbf{u}+\partial_z w)/2 - \partial_z(w\zeta)/2 - w\partial_z\zeta/2 - f\delta - \zeta\delta - \mathbf{e}_z\cdot\nabla w\times\partial_z\mathbf{u} + \mathbf{e}_z\cdot\nabla\times\mathcal{D}_\mathbf{u}$$

$$\partial_t\delta = -(\mathbf{u}\cdot\nabla)\delta - \delta(\nabla\cdot\mathbf{u})/2 + \delta(\nabla\cdot\mathbf{u}+\partial_z w)/2 - \partial_z(w\delta)/2 - w\partial_z\delta/2 + f\zeta - \left(\delta^2 - 2J(u,v)\right) - c_p\bar{\theta}\nabla^2\pi' - \nabla w\cdot\partial_z\mathbf{u} + \nabla\cdot\mathcal{D}_\mathbf{u}$$

$$\partial_t\Phi = -\nabla\cdot(\mathbf{u}\Phi) - \partial_z(w\Phi) + \Phi D_3 \underbrace{- f\zeta\delta}_{\text{Conversion}} \underbrace{- \zeta\zeta\delta}_{\text{Stretching}} \underbrace{- \zeta\mathbf{e}_z\cdot\nabla w\times\partial_z\mathbf{u}}_{\text{Tilting}} \underbrace{+ \zeta\mathbf{e}_z\cdot\nabla\times\mathcal{D}_\mathbf{u}}_{\text{Diffusion}}$$

$$\partial_t\Psi = -\nabla\cdot(\mathbf{u}\Psi) - \partial_z(w\Psi) + \Psi D_3 \underbrace{+ f\delta\zeta}_{\text{Conversion}} \underbrace{- \delta\left(\delta^2 - 2J(u,v)\right)}_{\text{Stretching}} \underbrace{- c_p\bar{\theta}\delta\nabla^2\pi'}_{\text{Pressure-related}} \underbrace{- \delta\nabla w\cdot\partial_z\mathbf{u}}_{\text{Tilting}} \underbrace{+ \delta\nabla\cdot\mathcal{D}_\mathbf{u}}_{\text{Diffusion}}$$

$$\Phi(\mathbf{k}) = \frac{\bar{\rho}_d}{2}(\zeta,\zeta)_\mathbf{k} \qquad \Psi(\mathbf{k}) = \frac{\bar{\rho}_d}{2}(\delta,\delta)_\mathbf{k}$$

$$\partial_t\Phi(\mathbf{k}) = \bar{\rho}_d(\zeta,\partial_t\zeta)_\mathbf{k} \qquad \partial_t\Psi(\mathbf{k}) = \bar{\rho}_d(\delta,\partial_t\delta)_\mathbf{k}$$

$$\partial_t\Phi(\mathbf{k}) = T_\zeta(\mathbf{k}) + \partial_z F_{\zeta\uparrow}(\mathbf{k}) + C_{\delta\to\zeta}(\mathbf{k}) + \alpha_\zeta(\mathbf{k}) + \beta_\zeta(\mathbf{k}) + \varepsilon_\zeta(\mathbf{k}) + D_\zeta(\mathbf{k})$$

$$\partial_t\Psi(\mathbf{k}) = T_\delta(\mathbf{k}) + \partial_z F_{\delta\uparrow}(\mathbf{k}) - C_{\delta\to\zeta}(\mathbf{k}) + \alpha_\delta(\mathbf{k}) + \beta_\delta(\mathbf{k}) + P_\delta(\mathbf{k}) + \varepsilon_\delta(\mathbf{k}) + D_\delta(\mathbf{k})$$

$$T_\zeta(\mathbf{k}) = -\bar{\rho}_d(\zeta,\mathbf{u}\cdot\nabla\zeta + \zeta\nabla\cdot\mathbf{u}/2)_\mathbf{k} + \bar{\rho}_d\left[(\partial_z\zeta,w\zeta)_\mathbf{k} - (\zeta,w\partial_z\zeta)_\mathbf{k}\right]/2$$

$$T_\delta(\mathbf{k}) = -\bar{\rho}_d(\delta,\mathbf{u}\cdot\nabla\delta + \delta\nabla\cdot\mathbf{u}/2)_\mathbf{k} + \bar{\rho}_d\left[(\partial_z\delta,w\delta)_\mathbf{k} - (\delta,w\partial_z\delta)_\mathbf{k}\right]/2$$

$$F_{\zeta\uparrow}(\mathbf{k}) = -\bar{\rho}_d(\zeta,w\zeta)_\mathbf{k}/2 \qquad F_{\delta\uparrow}(\mathbf{k}) = -\bar{\rho}_d(\delta,w\delta)_\mathbf{k}/2$$

$$C_{\delta\to\zeta}(\mathbf{k}) = -f\bar{\rho}_d(\zeta,\delta)_\mathbf{k}$$

$$\alpha_\zeta(\mathbf{k}) = -\bar{\rho}_d(\zeta,\zeta\delta)_\mathbf{k} \qquad \alpha_\delta(\mathbf{k}) = -\bar{\rho}_d\left(\delta,\delta^2 - 2J(u,v)\right)_\mathbf{k}$$

$$\beta_\zeta(\mathbf{k}) = -\bar{\rho}_d(\zeta,\mathbf{e}_z\cdot\nabla w\times\partial_z\mathbf{u})_\mathbf{k} \quad \beta_\delta(\mathbf{k}) = -\bar{\rho}_d(\delta,\nabla w\cdot\partial_z\mathbf{u})_\mathbf{k}$$

$$P_\delta(\mathbf{k}) = -c_p\bar{\rho}_d\bar{\theta}(\delta,\nabla^2\pi')_\mathbf{k} = |\mathbf{k}|^2 c_p\bar{\rho}_d\bar{\theta}(\nabla\cdot\mathbf{u},\pi')_\mathbf{k}$$

$$\varepsilon_\zeta(\mathbf{k}) = \bar{\rho}_d\left\{(\zeta,\zeta(\partial_z w + \nabla\cdot\mathbf{u}))_\mathbf{k} + \partial_z\ln\bar{\rho}_d(\zeta,w\zeta)_\mathbf{k}\right\}/2$$

$$\varepsilon_\delta(\mathbf{k}) = \bar{\rho}_d\left\{(\delta,\delta(\partial_z w + \nabla\cdot\mathbf{u}))_\mathbf{k} + \partial_z\ln\bar{\rho}_d(\delta,w\delta)_\mathbf{k}\right\}/2$$

$$D_\zeta(\mathbf{k}) = \bar{\rho}_d(\zeta,\mathbf{e}_z\cdot\nabla\times\mathcal{D}_\mathbf{u})_\mathbf{k} \qquad D_\delta(\mathbf{k}) = \bar{\rho}_d(\delta,\nabla\cdot\mathcal{D}_\mathbf{u})_\mathbf{k}$$

2. Methodology and Idealized Simulation

涡度/散度谱收支方程推导

能量串级与涡度/散度平方串级的显式关系表达式

$$T_h(\mathbf{k}) = -\bar{\rho}_d\left(\mathbf{u},\ \mathbf{u}\cdot\nabla\mathbf{u}+\mathbf{u}\nabla\cdot\mathbf{u}/2\right)_\mathbf{k} + \bar{\rho}_d\left[\left(\partial_z\mathbf{u},\ w\mathbf{u}\right)_\mathbf{k} - \left(\mathbf{u},\ w\partial_z\mathbf{u}\right)_\mathbf{k}\right]/2$$

Peng et al. *JAS* 2015a

$$T_\zeta(\mathbf{k}) = -\bar{\rho}_d\left(\zeta,\mathbf{u}\cdot\nabla\zeta+\zeta\nabla\cdot\mathbf{u}/2\right)_\mathbf{k} + \bar{\rho}_d\left[\left(\partial_z\zeta,w\zeta\right)_\mathbf{k} - \left(\zeta,w\partial_z\zeta\right)_\mathbf{k}\right]/2$$

$$T_\delta(\mathbf{k}) = -\bar{\rho}_d\left(\delta,\mathbf{u}\cdot\nabla\delta+\delta\nabla\cdot\mathbf{u}/2\right)_\mathbf{k} + \bar{\rho}_d\left[\left(\partial_z\delta,w\delta\right)_\mathbf{k} - \left(\delta,w\partial_z\delta\right)_\mathbf{k}\right]/2$$

$$\left(\delta,\nabla\cdot\boldsymbol{\varphi}\right)_\mathbf{k} + \left(\zeta,\mathbf{e}_z\cdot\nabla\times\boldsymbol{\varphi}\right)_\mathbf{k} = |\mathbf{k}|^2\left(\mathbf{u},\boldsymbol{\varphi}\right)_\mathbf{k}$$

$$\left(\phi,\nabla^2\varphi\right)_\mathbf{k} = -|\mathbf{k}|^2\left(\phi,\varphi\right)_\mathbf{k}$$

$$|\mathbf{k}|^2 T_h(\mathbf{k}) = T_\zeta(\mathbf{k}) + T_\delta(\mathbf{k}) + \alpha_\zeta(\mathbf{k}) + \alpha_\delta(\mathbf{k}) + \beta_\zeta(\mathbf{k}) + \beta_\delta(\mathbf{k}) + Res(\mathbf{k})$$

$$\nabla\cdot\left(\mathbf{u}\cdot\nabla\mathbf{u}\right) = \delta^2 - 2J\left(u,v\right) + \mathbf{u}\cdot\nabla\delta$$

$$\nabla\cdot\left(\mathbf{u}\nabla\cdot\mathbf{u}/2\right) = \delta\nabla\cdot\mathbf{u}/2 + \mathbf{u}\cdot\nabla\delta/2$$

$$\nabla\times\left(\mathbf{u}\cdot\nabla\mathbf{u}\right) = \delta\zeta + \mathbf{u}\cdot\nabla\zeta$$

$$\nabla\times\left(\mathbf{u}\nabla\cdot\mathbf{u}/2\right) = \zeta\nabla\cdot\mathbf{u}/2 - \mathbf{u}\times\nabla\delta/2$$

$$Res(\mathbf{k}) = \bar{\rho}_d\left[\left(\zeta,\mathbf{e}_z\cdot\mathbf{u}\times\nabla\left(\delta+\partial_z w\right)\right)_\mathbf{k} - \left(\delta,\mathbf{u}\cdot\nabla\left(\delta+\partial_z w\right)\right)_\mathbf{k}\right]/2 + \bar{\rho}_d\partial_z\left[\left(\zeta,\mathbf{e}_z\cdot\nabla w\times\mathbf{u}\right)_\mathbf{k} + \left(\delta,\mathbf{u}\cdot\nabla w\right)_\mathbf{k}\right]/2$$

$$\nabla\cdot\mathbf{u} + \partial_z w + w\partial_z\ln\bar{\rho}_d = 0$$

$$Res(\mathbf{k}) = \partial_z\left[\bar{\rho}_d\left(\zeta,\mathbf{e}_z\cdot\nabla w\times\mathbf{u}\right)_\mathbf{k} + \bar{\rho}_d\left(\delta,\mathbf{u}\cdot\nabla w\right)_\mathbf{k}\right]/2$$

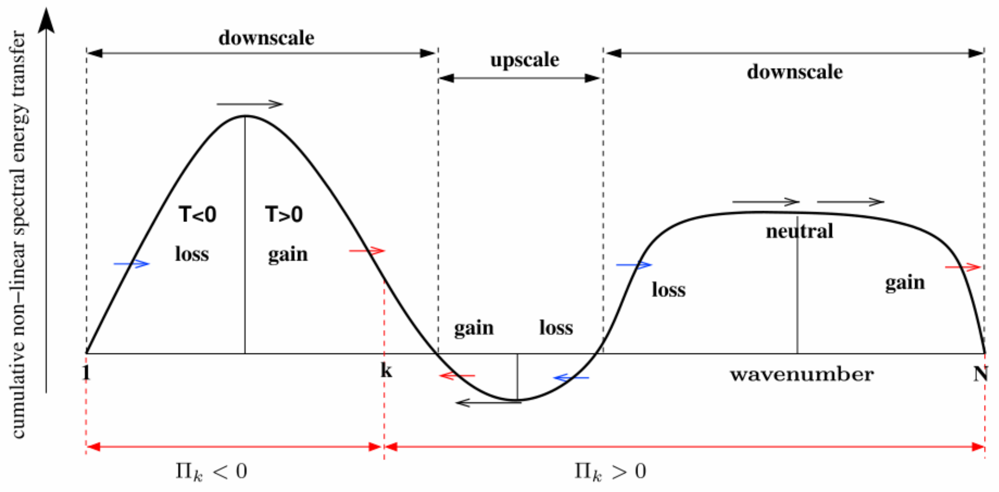
2. Methodology and Idealized Simulation

涡度/散度谱收支方程推导

非线性谱通量

$$\Pi_T^{\zeta}\left[k_h\right]=\sum_{|\mathbf{k}|\ge k_h} T_{\zeta}(\mathbf{k})$$

$$\Pi_T^{\delta}\left[k_h\right]=\sum_{|\mathbf{k}|\ge k_h} T_{\delta}(\mathbf{k})$$



收支方程累积形式

$$\partial_t\Phi(\mathbf{k})=T_{\zeta}(\mathbf{k})+\partial_z F_{\zeta\uparrow}(\mathbf{k})+C_{\delta\to\zeta}(\mathbf{k})+\alpha_{\zeta}(\mathbf{k})+\beta_{\zeta}(\mathbf{k})+\varepsilon_{\zeta}(\mathbf{k})+D_{\zeta}(\mathbf{k})$$

$$\partial_t\Psi(\mathbf{k})=T_{\delta}(\mathbf{k})+\partial_z F_{\delta\uparrow}(\mathbf{k})-C_{\delta\to\zeta}(\mathbf{k})+\alpha_{\delta}(\mathbf{k})+\beta_{\delta}(\mathbf{k})+P_{\delta}(\mathbf{k})+\varepsilon_{\delta}(\mathbf{k})+D_{\delta}(\mathbf{k})$$

$$\Pi_{\Phi}\left[k_h\right]=\sum_{|\mathbf{k}|\ge k_h}\Phi(\mathbf{k})$$

$$\Pi_{F\uparrow}^{(\,)}\left[k_h\right]=\sum_{|\mathbf{k}|\ge k_h} F_{(\,)\uparrow}(\mathbf{k})$$

$$\partial_t\Pi_{\Phi}\left[k_h\right]=\Pi_T^{\zeta}\left[k_h\right]+\partial_z\Pi_{F\uparrow}^{\zeta}\left[k_h\right]+\Pi_C\left[k_h\right]+\Pi_{\alpha}^{\zeta}\left[k_h\right]+\Pi_{\beta}^{\zeta}\left[k_h\right]+\Pi_{\varepsilon}^{\zeta}\left[k_h\right]+\Pi_D^{\zeta}\left[k_h\right]$$

$$\partial_t\Pi_{\Psi}\left[k_h\right]=\Pi_T^{\delta}\left[k_h\right]+\partial_z\Pi_{F\uparrow}^{\delta}\left[k_h\right]-\Pi_C\left[k_h\right]+\Pi_{\alpha}^{\delta}\left[k_h\right]+\Pi_{\beta}^{\delta}\left[k_h\right]+\Pi_P^{\delta}\left[k_h\right]+\Pi_{\varepsilon}^{\delta}\left[k_h\right]+\Pi_D^{\delta}\left[k_h\right]$$

2. Methodology and Idealized Simulation

理想干斜压波模拟

◆ **模式配置**

WRF3.2，*f* 平面，
周期通道 4000 ×10000 km
垂直方向 30 km，180 层
水平格距12.5 × 12.5 km
水平5阶、垂直3阶平流方案
垂直混合YSU边界层方案
垂直速度Reyleigh阻尼
不考虑湿过程、辐射过程、表面通量、边界层混合

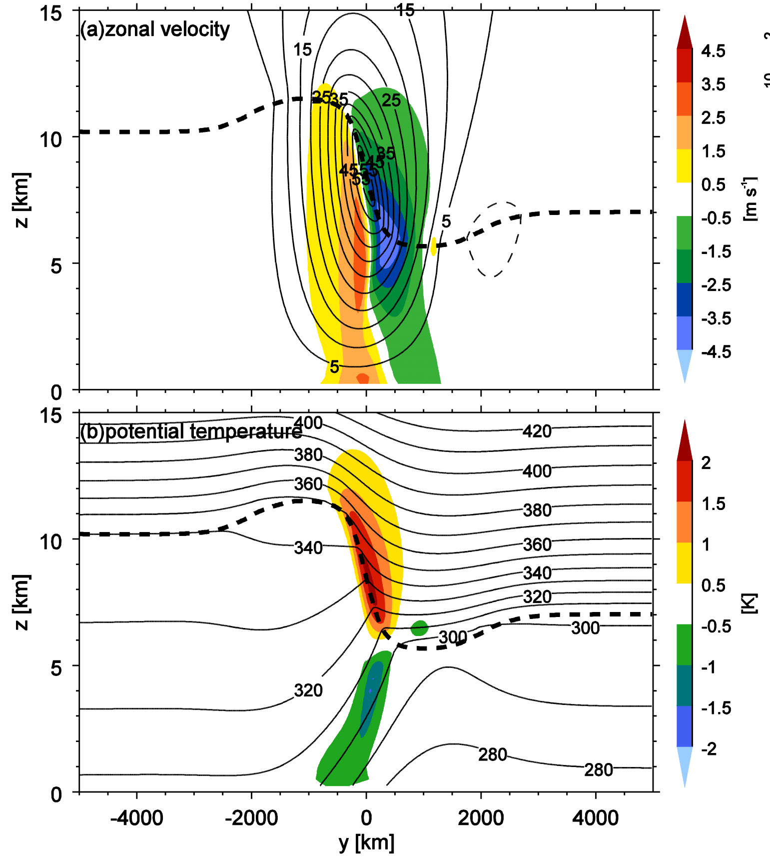
◆ **初始化 — 位涡反演**

强急流 U_{max}~58 m s^{-1}；
位涡反演得到的平衡场；
用增长最快的标准模扰动。

◆ **试验设计**

Dry

◆ **彩色阴影：最快增长标准模扰动**



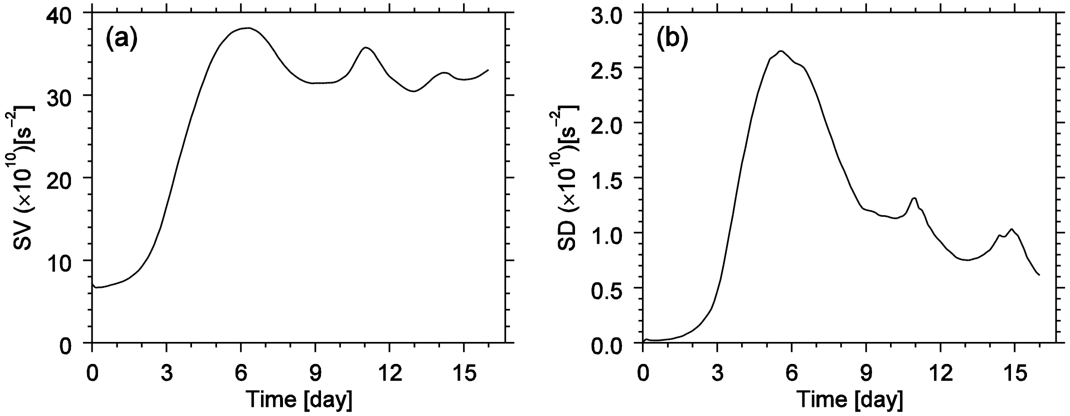
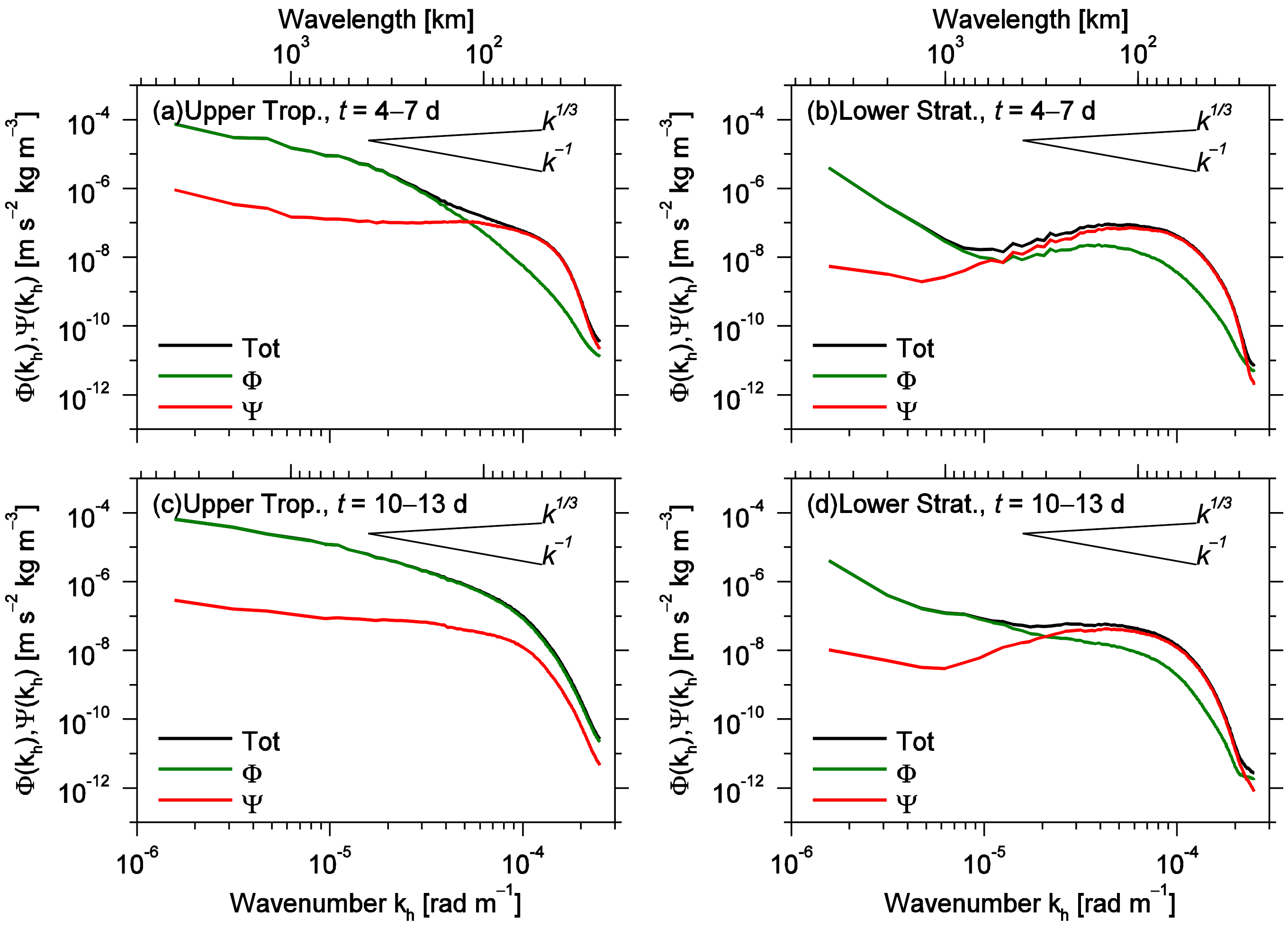


Fig. 2. Time series of mass-weighted average (a) squared vorticity (SV) and (b) squared divergence (SD) per unit mass. Here the mass-weighted average is computed over the entire domain.

Early phase: 4-7 days
Intermediate phase: 7-10 days
Late phase: 10-13 days

3. Vorticity/divergence Spectra and Spectral budgets

Vorticity/divergence Spectra



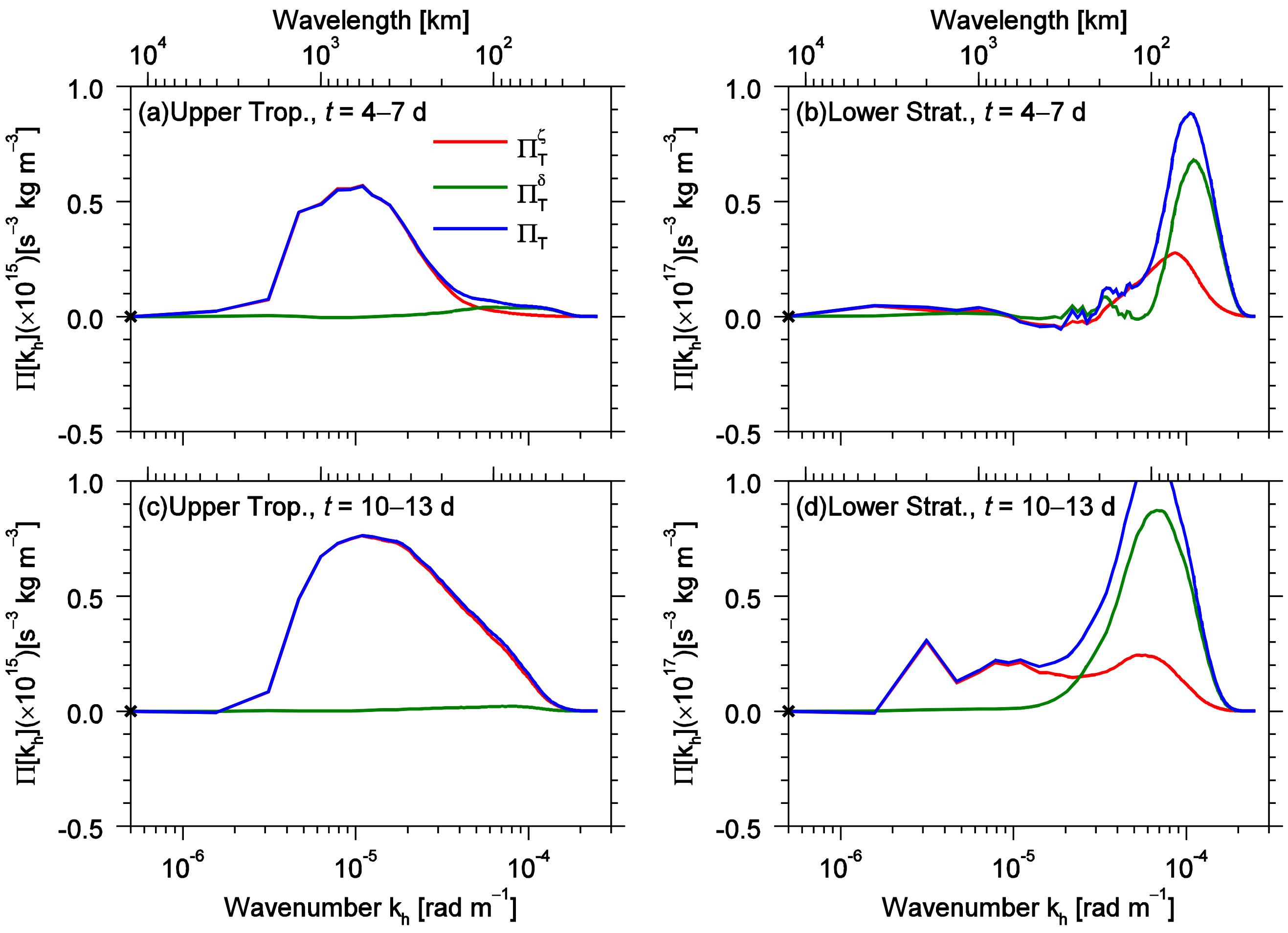
- In the upper troposphere, at both times, the SD spectrum is shallower than the SV spectrum; however, the amplitude of the former is much smaller than that of the latter, resulting in the lack of spectral transition in the total spectrum at this level.
- In the lower stratosphere, the SD spectrum crosses the SV spectrum in the mesoscale range over both time intervals and therefore a distinct spectral transition can be found in the total spectrum.
- The lower-stratospheric spectral transition is much more remarkable at the early phase, which should be related to the shortage of mesoscale squared vorticity around the transition scale of 500 km caused by the insufficient development of high-level weather systems.

3. Vorticity/divergence Spectra and Spectral budgets

Spectral budgets- Spectral transfer terms

$$T_\zeta(\mathbf{k}) = -\bar{\rho}_d(\zeta, \mathbf{u}\cdot\nabla\zeta + \zeta\nabla\cdot\mathbf{u}/2)_\mathbf{k} + \bar{\rho}_d\left[(\partial_z\zeta, w\zeta)_\mathbf{k} - (\zeta, w\partial_z\zeta)_\mathbf{k}\right]/2$$

$$T_\delta(\mathbf{k}) = -\bar{\rho}_d(\delta, \mathbf{u}\cdot\nabla\delta + \delta\nabla\cdot\mathbf{u}/2)_\mathbf{k} + \bar{\rho}_d\left[(\partial_z\delta, w\delta)_\mathbf{k} - (\delta, w\partial_z\delta)_\mathbf{k}\right]/2$$

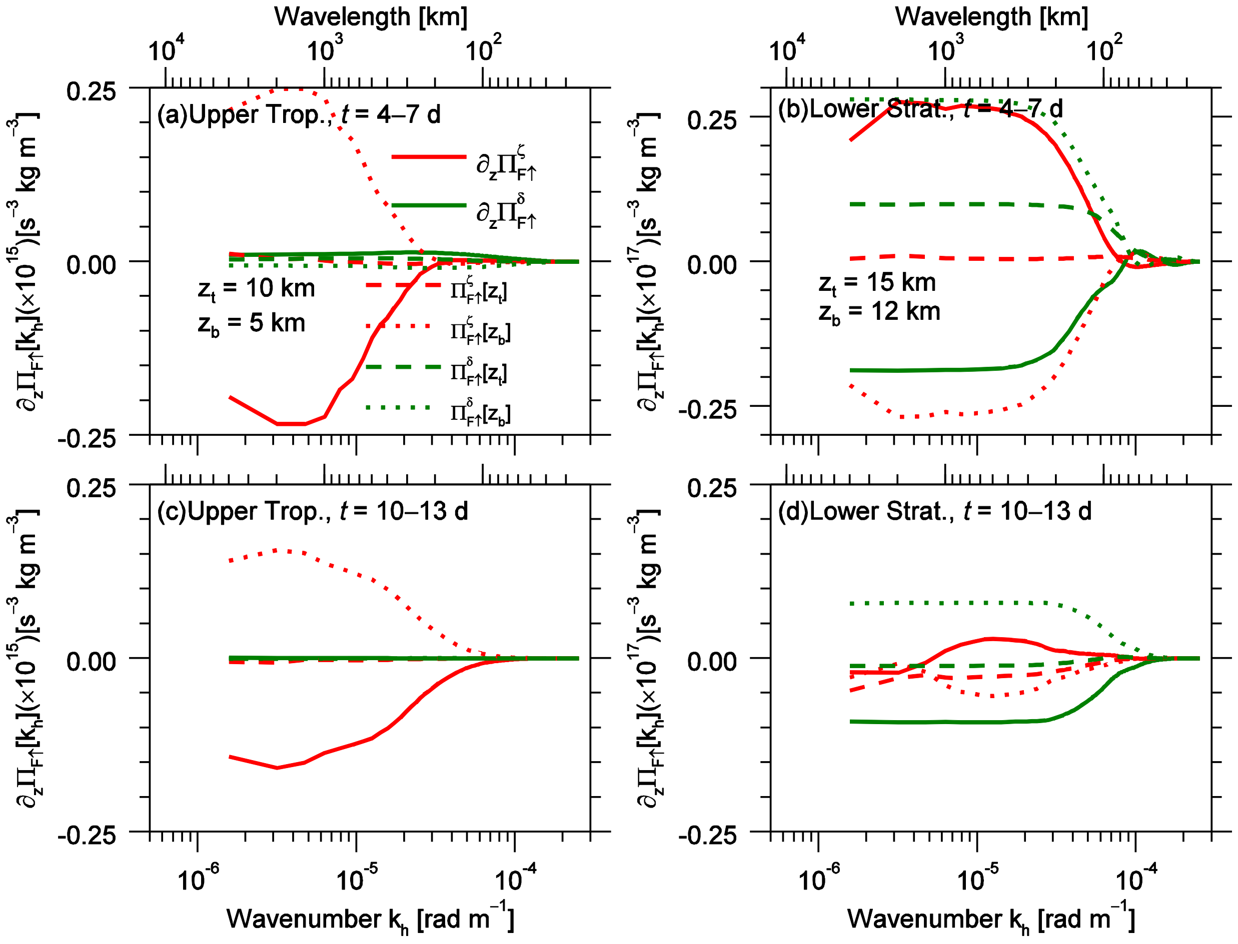


- In the dry baroclinic wave simulation the upper troposphere is almost completely dominated by the downscale SV transfer from synoptic scales to mesoscales;
- In contrast, the lower stratosphere is dominated by the downscale SV transfer at synoptic scales while by the downscale SD transfer at mesoscales.
- However, there is not an obvious subrange of wavenumbers over which the nonlinear flux of SV or SD is nearly constant, mainly because each of them is not conserved in this range of scales.

3. Vorticity/divergence Spectra and Spectral budgets

Spectral budgets- Spectral vertical fluxes

$F_{\zeta\uparrow}(\mathbf{k}) = -\bar{\rho}_d (\zeta, w\zeta)_{\mathbf{k}}/2$ $F_{\delta\uparrow}(\mathbf{k}) = -\bar{\rho}_d (\delta, w\delta)_{\mathbf{k}}/2$



$$\partial_z \Pi_{F\uparrow}[k_h] = (\Delta z)^{-1}\Pi_{F\uparrow}[k_h](z_t) - (\Delta z)^{-1}\Pi_{F\uparrow}[k_h](z_b)$$

$\partial_{k_h}\Pi_{F\uparrow}[k_h] > 0$ indicates an upward vertical flux.

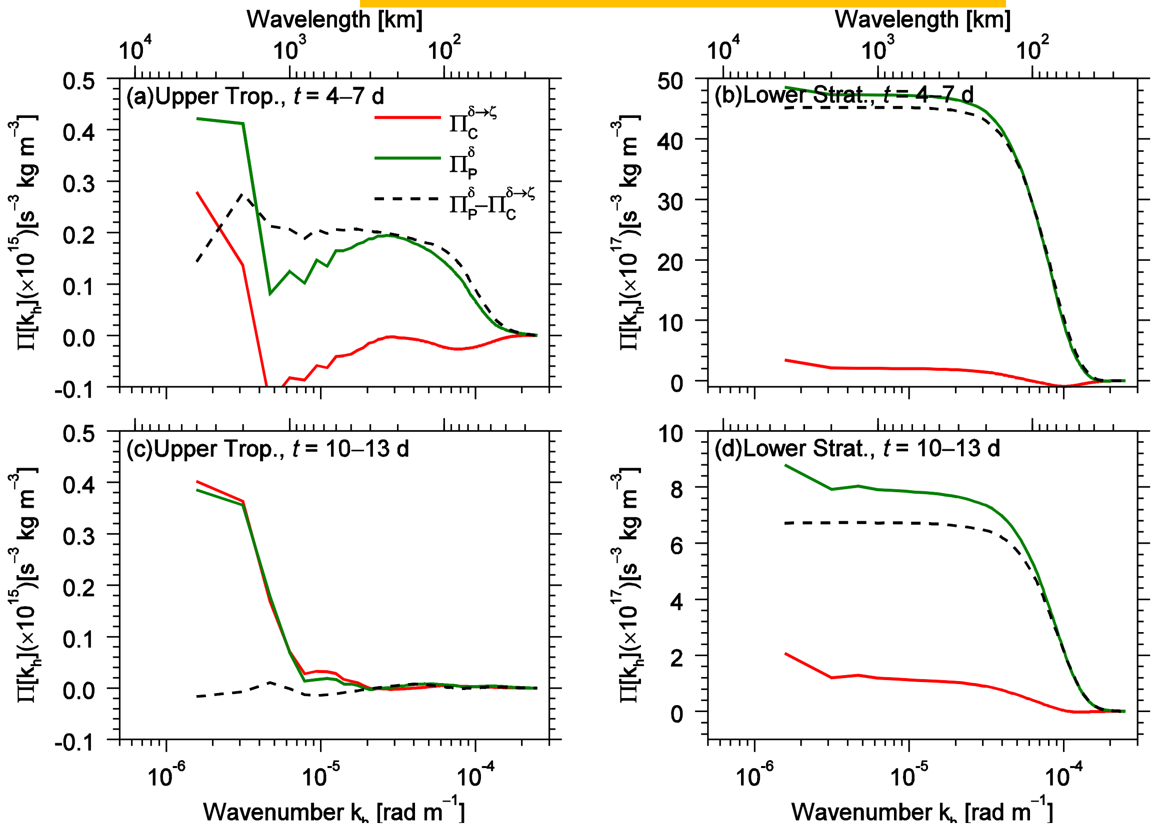
- In the upper troposphere, the vertical SV flux divergence tends to remove SV from this layer, mainly by the downward flux through the bottom of this layer; the divergence of the vertical SD flux is almost zero on all wavenumbers.
- In the lower stratosphere, the vertical SV flux divergence acts to increase SV, mainly by the upward flux through the bottom of this layer; the vertical SD flux divergence tends to remove SD during the early phase, mainly by the downward flux through the bottom of this layer.

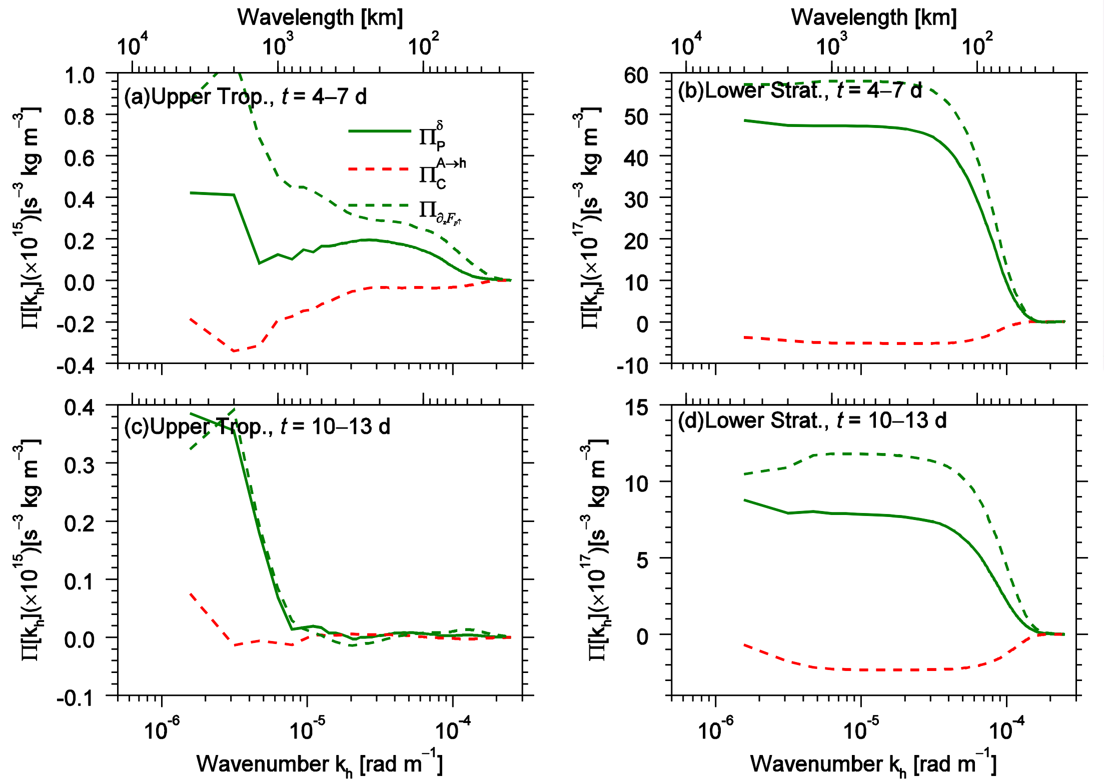
3. Vorticity/divergence Spectra and Spectral budgets

Spectral budgets- Spectral conversion and pressure-related fluxes

$C_{\delta\to\zeta}(\mathbf{k}) = -f\bar{\rho}_d(\zeta,\delta)_{\mathbf{k}}$ $P_\delta(\mathbf{k}) = -c_p\bar{\rho}_d\bar{\theta}(\delta,\nabla^2\pi')_{\mathbf{k}} = |\mathbf{k}|^2 c_p\bar{\rho}_d\bar{\theta}(\nabla\cdot\mathbf{u},\pi')_{\mathbf{k}}$

$P_\delta(\mathbf{k}) \approx |\mathbf{k}|^2\partial_z F_{p\uparrow}(\mathbf{k}) + |\mathbf{k}|^2 C_{A\to h}(\mathbf{k}) + |\mathbf{k}|^2 H_h(\mathbf{k})$

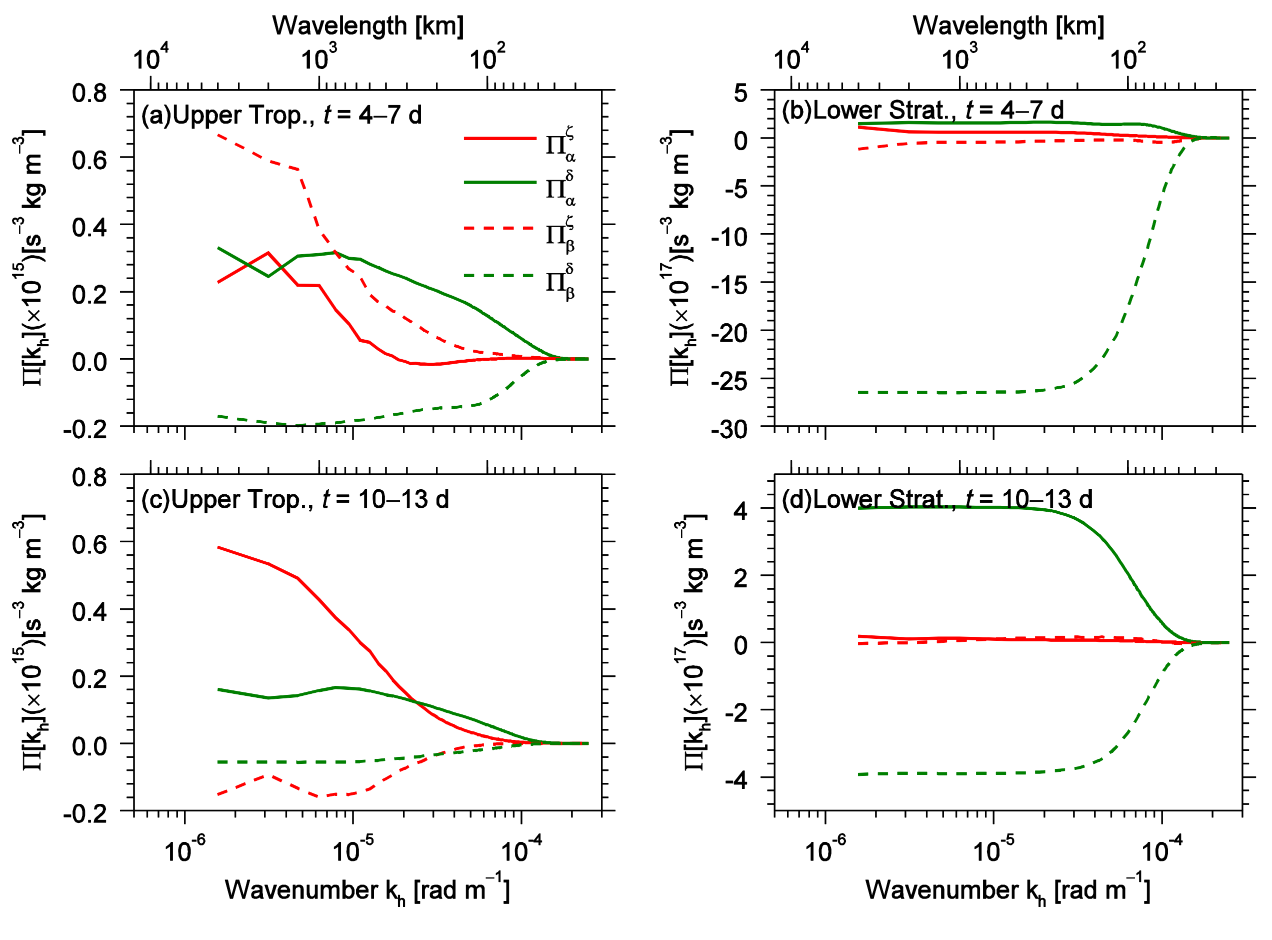




- In the upper troposphere, there exists an overall balance between the pressure-related term and the conversion term in the spectral SD budge. As a result, the net contribution between them at mesoscales is relatively weak.
- In the lower stratosphere, the net positive contribution between them on SD is much more significant at the small-scale end of mesoscales.
- The strong positive contribution of the pressure-related term on SD at the small-scale end of mesoscales in the lower stratosphere is mainly due to the vertically propagating IGWs

3. Vorticity/divergence Spectra and Spectral budgets

Spectral budgets- Spectral stretching and tilting terms



$\alpha_\zeta(\mathbf{k}) = -\bar{\rho}_d(\zeta, \zeta\delta)_\mathbf{k}$ $\quad$ $\alpha_\delta(\mathbf{k}) = -\bar{\rho}_d\left(\delta, \delta^2 - 2J(u,v)\right)_\mathbf{k}$

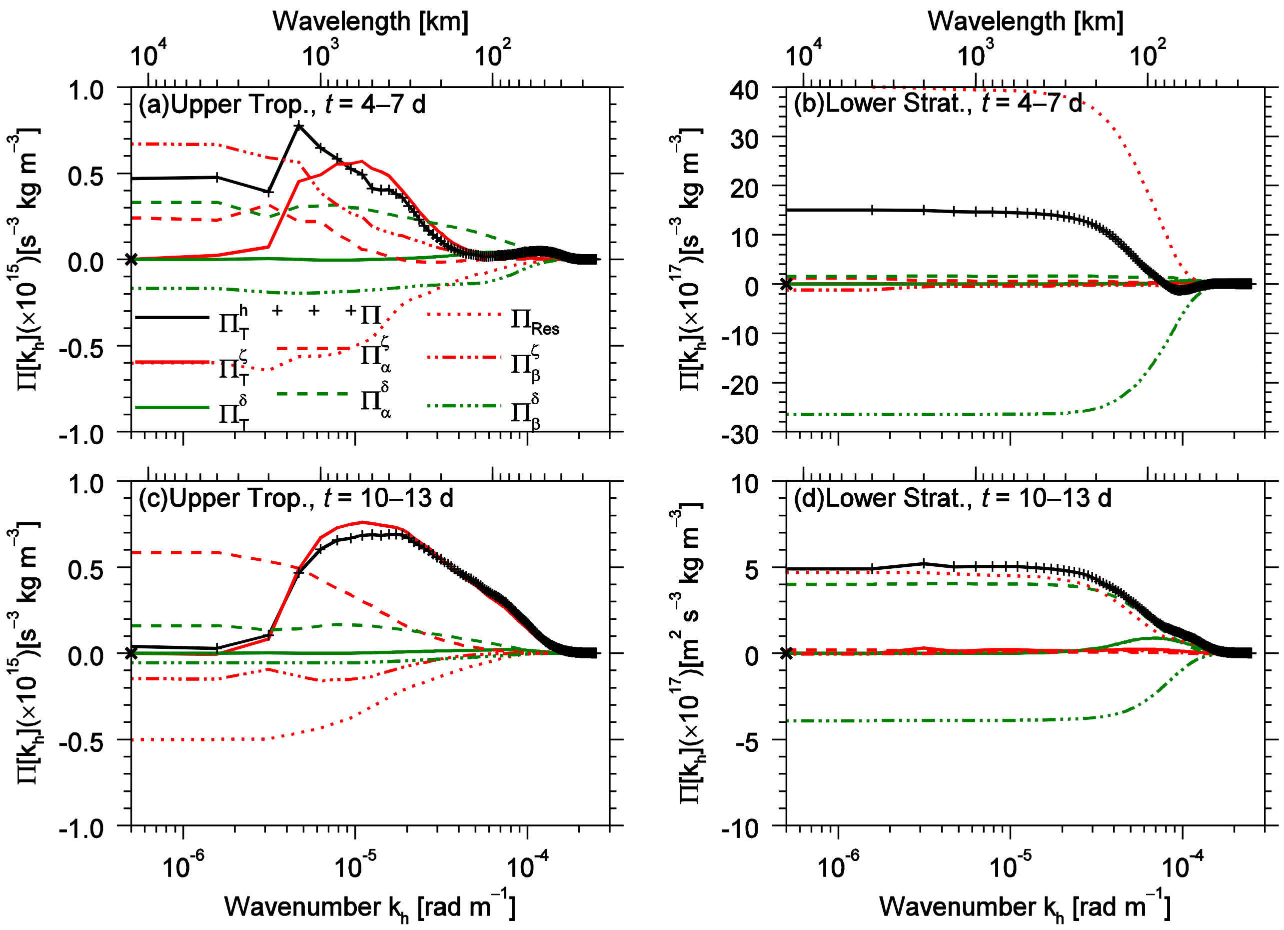
$\beta_\zeta(\mathbf{k}) = -\bar{\rho}_d(\zeta, \mathbf{e}_z \cdot \nabla w \times \partial_z \mathbf{u})_\mathbf{k}$ $\quad$ $\beta_\delta(\mathbf{k}) = -\bar{\rho}_d(\delta, \nabla w \cdot \partial_z \mathbf{u})_\mathbf{k}$

- These deformation terms have contributions comparable to or even much stronger than that of the corresponding cascade at mesoscales.
- The mesoscale atmosphere should be more like 3D turbulence rather than 2D turbulence, in agreement with the findings in many other studies before (e.g., Kafiabad and Bartello 2016, 2018).

4. Associating spectral energy and SV/SD transfers

Associating spectral energy and SV/SD transfers

$$|\mathbf{k}|^2 T_h(\mathbf{k}) = T_\zeta(\mathbf{k}) + T_\delta(\mathbf{k}) + \alpha_\zeta(\mathbf{k}) + \alpha_\delta(\mathbf{k}) + \beta_\zeta(\mathbf{k}) + \beta_\delta(\mathbf{k}) + Res(\mathbf{k})$$



- Essentially both spectral transfers and spectral stretching and tilting terms of SV and SD act as the physical processes governing energy transfer.
- In the upper troposphere, the downscale energy cascade is mainly governed by the downscale SV transfer at both synoptic scales and mesoscales and sometimes it is also significantly influenced by the stretching term on SV at synotic scales;
- In the lower stratosphere, the downscale energy cascade is mainly governed by the residual term related to 3D divergence and non-uniformly distributed vertical velocity, and sometimes it is also significantly influenced by the tilting term on SD.

5. Summary

- **A new formulation of the spectral budget of vertical vorticity and horizontal divergence suitable for the mesoscale atmosphere on an f plane is derived. Three main improvements:**
  - (i) both the squared vorticity (SV; i.e., enstrophy as usual) and squared divergence (SD) spectra are taken into account,
  - (ii) the spectral transfers of SV and SD between scales are exactly constructed under the nonlinear advection of the full horizontal velocity, and
  - (iii) the general relationship between spectral energy and SV/SD transfers is derived.
- **With this new formulation, the atmospheric spectra of divergent and rotational motion components are investigated through numerical simulation of idealized dry baroclinic waves.**
  - The upper troposphere is almost completely dominated by the downscale SV transfer at all scales, while the lower stratosphere is dominated by the downscale SV transfer at synoptic scales and by the downscale SD transfer at mesoscales.
  - The pressure-related term is largely cancelled out by the conversion term between SV and SD at both levels, but at the small-scale end of lower-stratospheric mesoscales there exists a significant net positive forcing, accounting for the distinct spectral transition of the total spectrum there.
- **An explicit association between spectral energy and SV/SD transfers is further made.**
  - In the upper troposphere, the downscale energy cascade is mainly governed by the downscale SV transfer; while in the lower stratosphere, it is mainly governed by the residual term related to non-uniformly distributed vertical velocity.

THANK YOU

Peng Jun, Zongheng Li, Lifeng Zhang, et al., 2023: Mesoscale Spectra of Vertical Vorticity and Horizontal Divergence in an Idealized Baroclinic Wave Simulation. *Journal of the Atmospheric Sciences*. doi: 10.1175/JAS-D-22-0213.1.