Key Laboratory of Cloud-Precipitation Physics and Severe Storms, Chinese Academy of Sciences

A New Triple-Moment Scheme (IAP-LACS) in Condensation and Warm Rain Formation Processes

Jun Zhang[1], Jiming Sun[1], Wei Deng[1], Mengqi Shao[1], Yuxia Ma[2]

1. Laboratory of Cloud-Precipitation Physics and Severe Storms, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing, China

2. College of atmospheric sciences, Lanzhou University, Lanzhou, Gansu, China

Introduction

Water vapor condensation growth and warm rain formation are the important processes for precipitation. However cloud droplet spectra are always spuriously broadened in condensation simulations with double-moment schemes. There are also large errors in the calculations of autoconversion rate and accretion rate with the current parameterization schemes. It makes the correct numerical simulations of condensation and warm rain formation become our big challenges for weather simulations with cloud resolving models and general circulation models. A new triple-moment scheme (IAP-LACS) in condensation and warm rain formation processes is presented to accurately describe the growth of a population of cloud drops. The three parameters-the shape parameter (α), the slope parameter (β) and the intercept parameter (N_0) in the newly developed triple-moment scheme are related to the three moments-the number concentration (M_0) , the water content (M_1) and the reflectivity (M_2) of drops. The analytical solutions of autoconversion and accretion rates are obtained.

Mathematical formulations in condensation process

$$f(m)=N_0 m^{\alpha-1}e^{-\beta m}$$

$$\frac{dm}{dt}=k[(\frac{1}{6}\pi\rho_w)^{-\frac{1}{3}}Sm^{\frac{1}{3}}-2a+8(\frac{1}{6}\pi\rho_w)^{\frac{2}{3}}bm^{-\frac{2}{3}}]$$

$$\frac{dM_1}{dt}=\int_0^\infty\frac{dm}{dt}f(m)dm+\frac{dM_1}{dt}|_{nucleation}$$

$$=kM_0[(\frac{1}{6}\pi\rho_w)^{-\frac{1}{3}}S\frac{\Gamma(\alpha+\frac{1}{3})}{\Gamma(\alpha)\beta^{\frac{1}{3}}}-2a+(\frac{1}{6}\pi\rho_w)^{\frac{2}{3}}8b\frac{\Gamma(\alpha-\frac{2}{3})}{\Gamma(\alpha)\beta^{-\frac{2}{3}}}]+\frac{dM_1}{dt}|_{nucleation},$$

$$\frac{dM_2}{dt}=H_2\int_0^\infty 2m\frac{dm}{dt}f(m)dm+\frac{dM_2}{dt}|_{nucleation}$$

$$=2H_2kM_0[(\frac{1}{6}\pi\rho_w)^{-\frac{1}{3}}S\frac{\Gamma(\alpha+\frac{4}{3})}{\Gamma(\alpha)\beta^{\frac{4}{3}}}-2a\frac{\Gamma(\alpha+1)}{\Gamma(\alpha)\beta}+(\frac{1}{6}\pi\rho_w)^{\frac{2}{3}}8b\frac{\Gamma(\alpha+\frac{1}{3})}{\Gamma(\alpha)\beta^{\frac{1}{3}}}]+\frac{dM_2}{dt}|_{nucleation}$$

$$M_0=H_0\int_0^\infty N_0 m^{\alpha-1}e^{-\beta m}dm=\frac{N_0\Gamma(\alpha)}{\beta^\alpha},$$

$$M_1=H_1\int_0^\infty N_0 m^{\alpha}e^{-\beta m}dm=\frac{N_0\Gamma(\alpha+1)}{\beta^{\alpha+1}}=\frac{M_0\alpha}{\beta},$$

$$M_2=H_2\int_0^\infty N_0 m^{\alpha+1}e^{-\beta m}dm=\frac{H_2N_0\Gamma(\alpha+2)}{\beta^{\alpha+2}}=\frac{H_2M_0(\alpha+1)\alpha}{\beta^2}$$

$$\frac{d\alpha}{dt}=\frac{2\beta(\alpha+1)}{M_0}\frac{dM_1}{dt}-\frac{\beta^2}{H_2M_0}\frac{dM_2}{dt}-\frac{\alpha(\alpha+1)}{M_0}\frac{dM_0}{dt},$$

$$\frac{d\beta}{dt}=\frac{\beta^2(2\alpha+1)}{M_0\alpha}\frac{dM_1}{dt}-\frac{\beta^3}{H_2M_0\alpha}\frac{dM_2}{dt}-\frac{\alpha\beta}{M_0}\frac{dM_0}{dt}.$$

$$\frac{d\alpha}{dt}=4k[\frac{1}{3}(\frac{1}{6}\pi\rho_w)^{-\frac{1}{3}}S\frac{\Gamma(\alpha+\frac{1}{3})}{\Gamma(\alpha)}\beta^{\frac{2}{3}}-a\beta+\frac{20}{3}(\frac{1}{6}\pi\rho_w)^{\frac{2}{3}}b\frac{\Gamma(\alpha-\frac{2}{3})}{\Gamma(\alpha)}\beta^{\frac{5}{3}}]$$
$$+\frac{2\beta(\alpha+1)}{M_0}\frac{dM_1}{dt}|_{nucleation}-\frac{\beta^2}{H_2M_0}\frac{dM_2}{dt}|_{nucleation}-\alpha(\alpha+1)\frac{dlnM_0}{dt}|_{nucleation},$$

$$\frac{d\beta}{dt}=k[\frac{1}{3}(\frac{1}{6}\pi\rho_w)^{-\frac{1}{3}}S\frac{\Gamma(\alpha+\frac{1}{3})}{\Gamma(\alpha+1)}\beta^{\frac{5}{3}}-2a\frac{\beta^2}{\alpha}+\frac{56}{3}(\frac{1}{6}\pi\rho_w)^{\frac{2}{3}}b\frac{\Gamma(\alpha-\frac{2}{3})}{\Gamma(\alpha+1)}\beta^{\frac{8}{3}}]$$
$$+\frac{\beta^2(2\alpha+1)}{M_0\alpha}\frac{dM_1}{dt}|_{nucleation}-\frac{\beta^3}{H_2M_0\alpha}\frac{dM_2}{dt}|_{nucleation}-\alpha\beta\frac{dlnM_0}{dt}|_{nucleation}.$$

Analytical Solutions of Autoconversion (AUTO) Rate and Accretion (ACC) Rate

The increase and decrease of cloud droplets during AUTO process:

$$H_p\int_{x_m}^\infty x^p\frac{1}{2}\int_0^x K(y,x-y)f_c(y,t)f_c(x-y,t)dydx=H_p\frac{N_c^2k_c}{\Gamma(\alpha_c)}\frac{\Gamma(\alpha_c+2)}{\Gamma(2\alpha_c+2)}\frac{\Gamma(p+2\alpha_c+2,\beta_c x_m)}{\beta_c^{p+2}},$$

$$-H_p\int_{x_m}^\infty x^p\int_0^\infty K(y,x)f_c(y,t)f_c(x,t)dydx=-H_p\frac{N_c^2k_c}{\Gamma^2(\alpha_c)}\frac{1}{\beta_c^{p+2}}[\Gamma(p+\alpha_c+2,\beta_c x_m)\Gamma(\alpha_c)+\Gamma(p+\alpha_c,\beta_c x_m)\Gamma(\alpha_c+2)],$$

The increase and decrease of of raindrops during ACC process:

$$H_p\int_0^\infty x^p\frac{1}{2}\int_0^x K(y,x-y)f_c(y,t)f_r(x-y,t)dydx=H_p\frac{N_cN_rk_r\beta_c^{\alpha_c}}{2\Gamma(\alpha_c)\beta_r^{p+\alpha_c+1}}\sum_{n=0}^\infty\Gamma(p+\alpha_c+\alpha_r+n+1)\frac{\Gamma(\alpha_c+n)}{n!\Gamma(\alpha_c+\alpha_r+n)}(1-\frac{\beta_c}{\beta_r})^n,$$

$$-H_p\int_0^\infty x^p\int_0^\infty K(y,x)f_c(y,t)f_r(x,t)dydx=-H_p\frac{N_cN_rk_r\Gamma(p+\alpha_r)}{\Gamma(\alpha_r)\beta_r^p}(\frac{p+\alpha_r}{\beta_r}+\frac{\alpha_c}{\beta_c}),$$

The increase and decrease of cloud droplets during ACC process:

$$H_p\int_0^\infty x^p\frac{1}{2}\int_0^x K(y,x-y)f_r(y,t)f_c(x-y,t)dydx=H_p\frac{N_cN_rk_r\beta_r^{\alpha_r}}{2\Gamma(\alpha_r)\beta_c^{p+\alpha_r+1}}\sum_{n=0}^\infty\Gamma(p+\alpha_c+\alpha_r+n+1)\frac{\Gamma(\alpha_r+n)}{n!\Gamma(\alpha_c+\alpha_r+n)}(1-\frac{\beta_r}{\beta_c})^n,$$

$$-H_p\int_0^\infty x^p\int_0^\infty K(y,x)f_r(y,t)f_c(x,t)dydx=-H_p\frac{N_cN_rk_r\Gamma(p+\alpha_c)}{\Gamma(\alpha_c)\beta_c^p}[\frac{p+\alpha_c}{\beta_c}+\frac{\alpha_r}{\beta_r}].$$

Where three moments: the zeroth (P=0), the first (P=1) and the second (P=2) moments, respectively representing the number concentration, the water content and the reflectivity, K(x,y) is the collection kernel of a drop of mass x colliding with a drop of mass y (Long ,1974), x_m=5.23x10^{-7}g , $H_0=H_1=1$, $H_2=(\pi\rho_w/6)^{-2}$.

Condensation experiment with a constant supersaturation

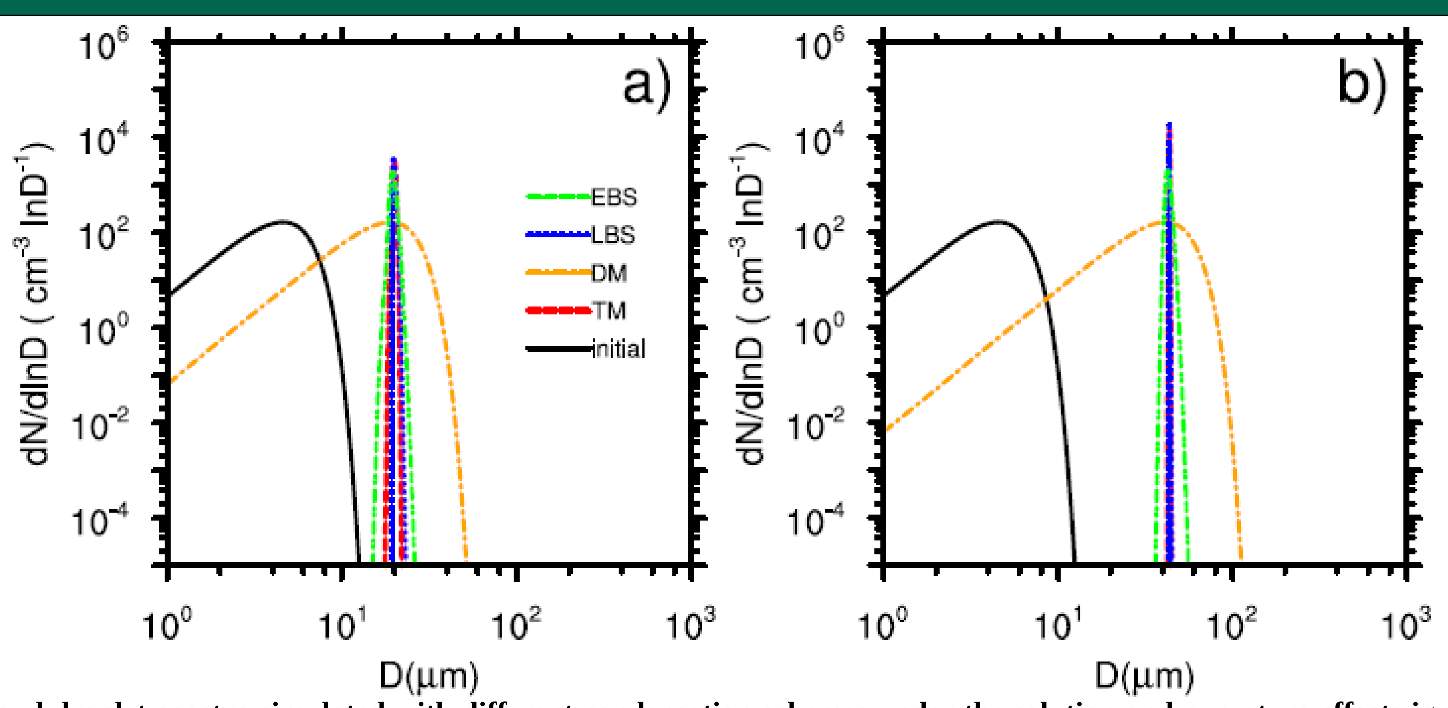

FIG. 1. Cloud droplet spectra simulated with different condensation schemes under the solution and curvature effects ignored (DM (the double-moment scheme), TM (the triple-moment scheme), LBS (the Lagrangian bin scheme) , EBS (the Eulerian-advection-based bin))

Fig.1 shows the evolution of cloud droplet spectrum with the newly developed triple-moment scheme（TM） matches that with the Lagrangian bin scheme (LBS) very well. The key point of the experiment is that the newly developed triple-moment scheme（TM） can be applied to accurately describe the diffusion growth of a population of cloud drops and even surpass Eulerian-advection-based bin（EBS） scheme in simulation without nucleation.

Numerical experiment for the Autoconversion (AUTO) Rate

Fig.2 shows comparisons of the six schemes in the calculations of the autoconversion rates with various initial cloud droplet number mixing ratios (30-2000cm^{-3}) and initial cloud droplet mass mixing ratios (0.25x10^{-6}-5x10^{-6}gcm^{-3}). It is generally agreed that the autoconversion rates increase with the increasing the cloud water content, but decrease with the increasing the number concentration. The simulated result of TM is closest to that of EBS

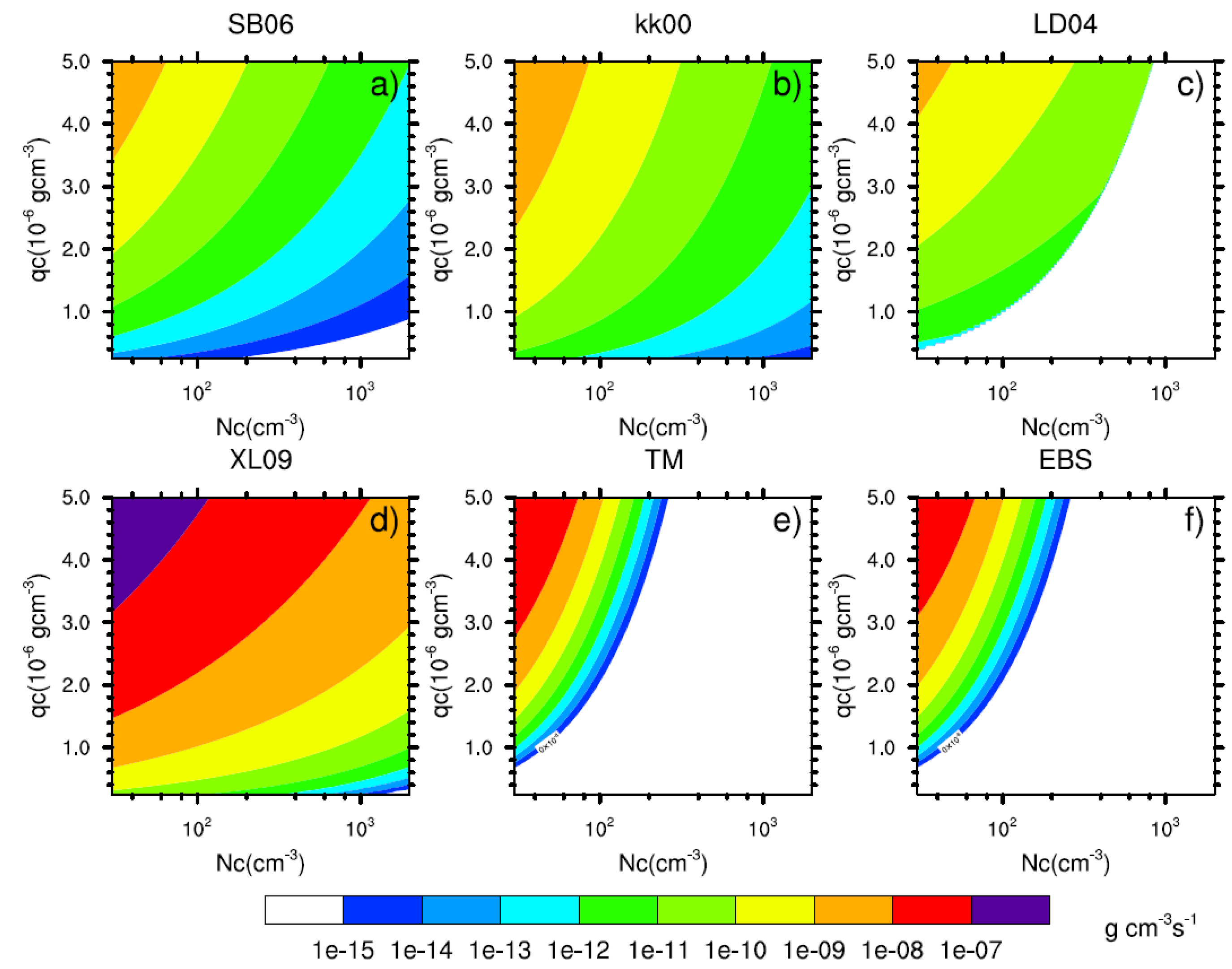

FIG. 2. The autoconversion rates of the cloud water content with various initial number concentration and initial water content of cloud droplets for different schemes: SB06 (Seifert and Beheng, 2006), kk00 (Khairoutdinov and Kogan, 2000), LD04 (Liu et al, 2004), XL09 (Xie and Liu, 2009), TM (the triple-moment scheme), EBS (the Eulerian-advection-based bin).

Numerical experiment for the new triple-moment scheme

Fig.3 shows that the evolutions of simulated spectra during condensation, coalescence and broken (Verlinde and Cotton,1993) with TM and DM compared to that with EBS. The simulated results of the new triple-moment scheme (TM) are closest to those of the Eulerian bin scheme (EBS).

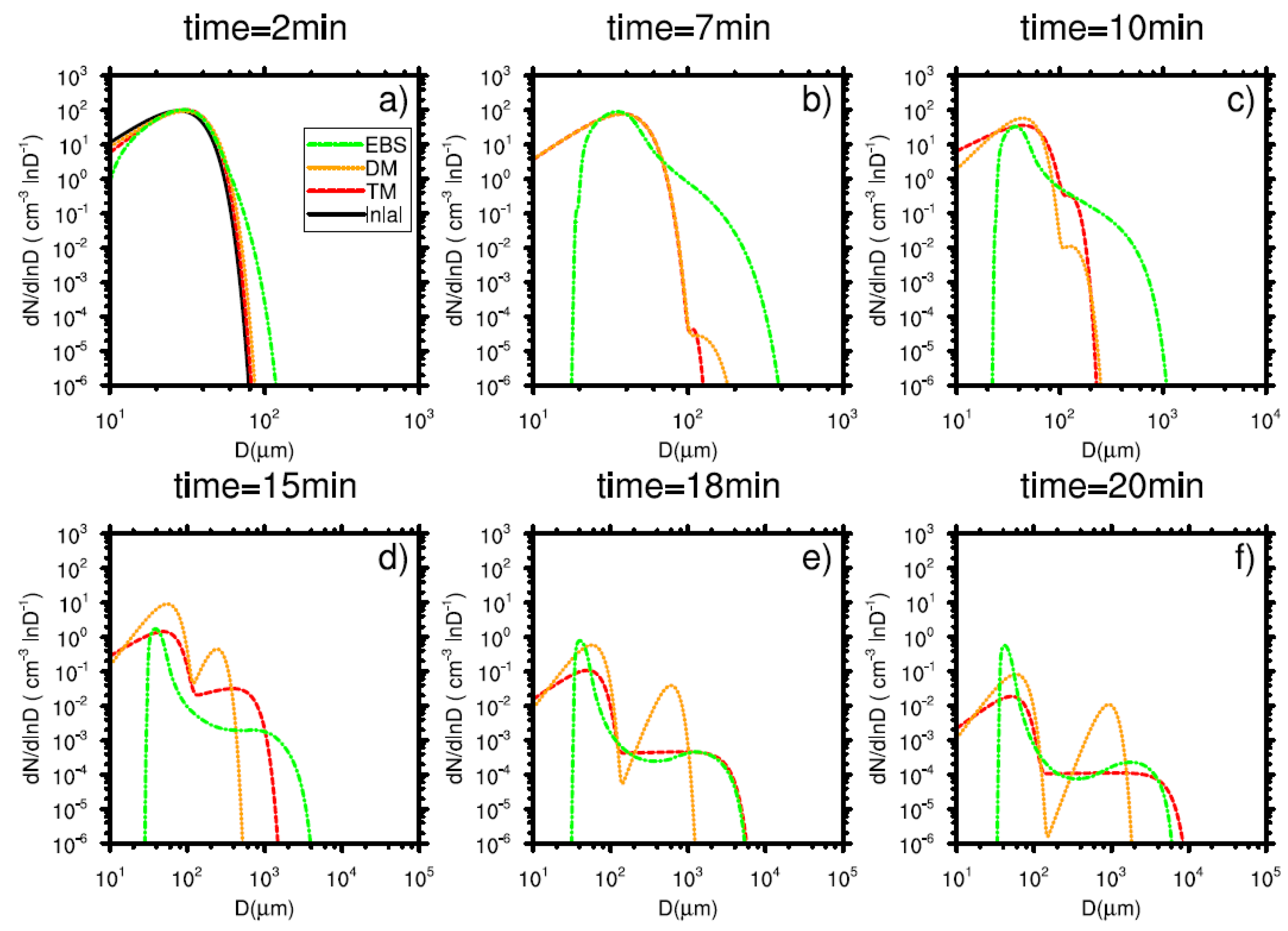

FIG. 3. Droplet spectra simulated with different schemes in the condensation, coalescence and broken processes during 20 minutes

Conclusions

In this study, we developed a new triple-moment scheme in condensation and warm rain formation processes based on the gamma distribution function with the size scale of the drop mass. Prognostic differential equations for the time-dependent slope and shape parameters have been established in the condensation process. The three parameters of gamma distribution for raindrops are obtained by performing analytical integrations of the microphysical transfer rates. The autoconversion and accretion rates are derived by solving the stochastic collection equation (SCE) with a parameterized collection kernel. These equations were derived by means of the number concentration, the water content and the reflectivity of drops. The simulated autoconversion rate with the new scheme is closest to that of the bin scheme compared the simulated results with the other parameterization schemes. The newly developed triple-moment scheme can be applied to accurately describe the growth of a population of cloud drops.

References

Sun, J., J. Zhang, W. Deng, W. Hu, and Y. Wang, 2021: Newly Developed Multiple-parameter Bulk Cloud Schemes. Part I: Cloud Droplet Nucleation Simulation and Parameterization. JAS., in revision.

J. Zhang., Sun, J, W. Deng, Y. Ma, 2021: Newly Developed Multiple-parameter Bulk Cloud Schemes. Part II: A New Triple-moment Condensation Scheme and Tests. JAS., in revision.

Khain, A. P., and Coauthors, 2015: Representation of microphysical processes in cloud-resolving models: Spectral (bin) microphysics versus bulk parameterization. Rev. Geophys., 53, 247–322, doi:10.1002/2014RG000468

Khairoutdinov, M. F., and Y. L. Kogan, 2000: A new cloud physics parameterization in a large eddy simulation model of marine stratocumulus. Monthly. Weather. Review., 128, 229–243.

Liu, Y. P., H. Daum, and R. McGraw, 2006: Parameterization of the autoconversion process. partII: Generalization of sundqvist-type parameterizations. Journal of the Atmospheric Sciences., 63, 1103–1109.

Long, A. B., 1974: Solutions to the droplet collection equation for polynomial kernels. J. Atmos. Sci., 31, 1040–1052.

Seifert, A., and K. D. Beheng, 2006: A two-moment cloud microphysics parameterization for mixed-phase clouds: Part I. model description. Meteorol. Atmos. Phys., 92, 45–66.

Verlinde, H., and W. R. Cotton, 1993: Fitting microphysical observations of nonsteady convective clouds to a numerical model: An application of the adjoint technique of data assimilation to a kinematic model. Mon. Wea. Rev., 121, 2776–2793

Xie, X., and X. Liu, 2009: Analytical three-moment autoconversion parameterization based on generalized gamma distribution. J. Geophys. Res., 114, D17 201